

Some Considerations on the Decoupling Control of TITO Systems

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Abstract - Proper understanding of the system nature is significant precondition for its successful control. Always actual issue about its mutual coupling was considered in this paper. Multivariable system with two-inputs and two-outputs (TITO) was in the focus here. Dominant pole placement method has been used in trying to tune PID controllers that should support decoupling control. The aim was to determine parameters of the PID controllers which, in combination with decoupler, can obtain good dynamical behavior of the system. Another goal was to simplify tuning procedure of PID controllers and enlarge possibility for introducing given approach into practice. But, research results indicate that proposed procedure leads to usage of P controllers, because they enable the best performances for the considered object. Research has been supported by simulations and therefore effectiveness of the proposed method, regarding quality of system behavior, was presented on the several examples.

Key words: Decoupling control, PID control, TITO process, dominant pole placement method

I INTRODUCTION

Multivariable systems are in focus of many surveys, in the recent decades. Their decoupling was researched intensively in [1-5]. Neither type of decoupler is universal. Which of them will provide appropriate compensation of mutual coupling depends on the object nature. In the present paper the static inverted decoupler has been used for the investigated object, like in [6]. Cantilever beam as an object of control has been taken into consideration. Its mathematical model was determined in [7]. Here electrohydraulic servosystem that was intended for structural testing was considered as a system with two inputs and two outputs (TITO). Decoupling control enables to take this kind of system as a finite number (in this case two) of SISO (single-input single-output) systems. Afterward, dominant pole placement method can be used for controller design, as one of the tuning rules. Das et al. [8] tune PID controllers by using guaranteed dominant pole placement method. Investigation of this method for the time delay systems was performed in [9-12]. Madady and Reza-Alikhani considered methods for first-order controller design using dominant pole placement, too [13]. Beside many other methods for PID controller tuning, Åström and Hägglund in [14] presented dominant pole placement method for several kind of objects. Filipović and Nedić [15] shown procedures for PI and PID controller design based on this method. Q.-G. Wang et al. [16] dealt with fourth-order object but without zeros. Extension of the original dominant pole placement

method for controller design to the multivariable systems is presented in [17-19].

In contrast to the aforementioned research, present paper deals with controller design for the TITO object, whose decoupled loops are of third-order with two left half plane zeros.

II SYSTEM DECOUPLING

General transfer function matrix of the considered object is given by (1).

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \quad (1)$$

Decoupling control strategy containing static inverted decoupler in the combination with PID controllers is shown in Fig. 1.

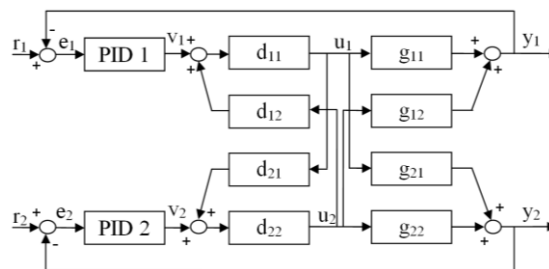


Fig. 1. Inverted decoupling control for TITO object [1].

According [5,6], decoupler was calculated using (2).

$$D(s) \Big|_{s=0} = \begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1.35 \\ 0.5 & 1 \end{bmatrix} \quad (2)$$

Apparent system (3), that should be obtained after decoupling, enables considering of the TITO system as a finite number of SISO systems.

$$Q(s) = G(s) \cdot D(s) = \begin{bmatrix} q_1(s) & 0 \\ 0 & q_2(s) \end{bmatrix} \quad (3)$$

Controllers will be designed based on diagonal elements of (3).

III CONTROLLER DESIGN

General expression for decentralized PID controller for the TITO process is given by (4), and its elements are presented with (5).

$$K(s) = \begin{bmatrix} k_1(s) & 0 \\ 0 & k_2(s) \end{bmatrix} \quad (4)$$

$$k_1(s) = K_{p1} + \frac{K_{i1}}{s} + K_{d1} \cdot s \quad (5)$$

$$k_2(s) = K_{p2} + \frac{K_{i2}}{s} + K_{d2} \cdot s$$

Where K_p , K_i and K_d are proportional, integral and derivative controller gains, respectively. In the inverted decoupling, controllers is designed for the diagonal elements of $Q(s)$ and hence: $q_1(s)=g_{11}(s)$ and $q_2(s)=g_{22}(s)$. Therefore, as previously stated, PID controller design using dominance pole placement method will be researched for the third-order transfer function with two left half plane zeros (6).

$$g_{ii}(s) = \frac{b_2s^2 + b_1s + b_0}{s^3 + a_2s^2 + a_1s + a_0} \quad (6)$$

According that, characteristic equation of the single loop is expressed by (7-9).

$$1 + g_{ii}(s) \cdot k_i(s) = 0 \quad (7)$$

$$1 + \frac{b_2s^2 + b_1s + b_0}{s^3 + a_2s^2 + a_1s + a_0} \cdot \frac{K_{d1} \cdot s^2 + K_{p1} \cdot s + K_{i1}}{s} = 0 \quad (8)$$

$$s \cdot (s^3 + a_2s^2 + a_1s + a_0) + (b_2s^2 + b_1s + b_0) \cdot (K_{d1} \cdot s^2 + K_{p1} \cdot s + K_{i1}) = 0 \quad (9)$$

Equation (10) is general form of the fourth-order characteristic equation. So, there are four poles: two conjugate complex (11) and two real. Since PID controller has three parameters, three dominant poles should be determined.

$$(s + \alpha\omega_n) \cdot (s + \beta\omega_n) \cdot (s^2 + 2\xi\omega_n s + \omega_n^2) = 0 \quad (10)$$

$$s = -\omega_n\xi + j\omega_n\sqrt{1 - \xi^2} \quad (11)$$

Here ω_n is natural frequency and ξ is damping coefficient.

Equalization of the (9) and (10) and large mathematical transformations lead to expressions for the PID controller gains (12).

$$K_p = \frac{(2\xi + \alpha + \beta)\omega_n - a_2}{b_2} \quad (12)$$

$$K_i = \frac{\alpha\beta\omega_n^4}{b_0}$$

$$K_d = \frac{(1 + 2\alpha\alpha + 2\beta\beta + \alpha\beta)b_1b_2\omega_n^2 - a_1b_1b_2}{b_0b_1b_2} - \frac{(\alpha + \beta + 2\alpha\alpha\beta)b_2^2\omega_n^3 + a_0b_2^2}{b_0b_1b_2} + \frac{(2\xi + \alpha + \beta)b_0b_2\omega_n - a_2b_0b_2}{b_0b_1b_2} - \frac{(2\xi + \alpha + \beta)b_1^2\omega_n + a_2b_1^2}{b_0b_1b_2}$$

IV EXAMPLES

Proposed procedure is illustrated through the three examples that have been explored to check its sensitivity to the model uncertainties and at the same time to investigate its applicability to the different objects.

Example 1.

Electrohydraulic servosystem for structural testing is shown in Fig. 2. Its mathematical model is given by (13) [7]. Control system serves to enable defined load to the cantilever beam. Intensity and character of the forces are characteristics that should be controlled by flow rates through the servovalves. Forces F_{r1} and F_{r2} are reference values. Values F_1 and F_2 from their transducers are object outputs.

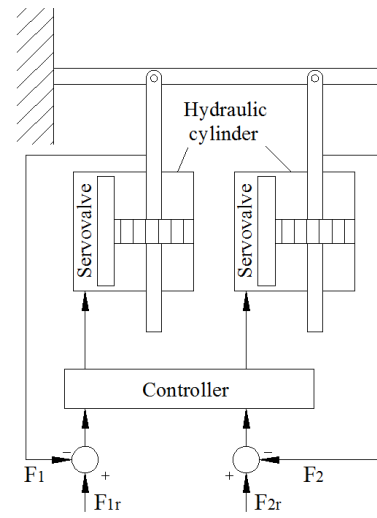


Fig. 2. Scheme of the double actuator electrohydraulic servosystem for structural testing [7].

$$G(s) = \frac{1}{\Delta(s)} \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

$$g_{11}(s) = 2.926 \cdot 10^2 s^4 + 1.9152 \cdot 10^4 s^3 +$$

$$+ 1.2667 \cdot 10^7 s^2 + 5.5825 \cdot 10^7 s + 4.7959 \cdot 10^9$$

$$g_{12}(s) = -3.8382 \cdot 10^4 s^3 - 1.7068 \cdot 10^7 s^2 -$$

$$- 8.3584 \cdot 10^7 s - 6.4967 \cdot 10^9$$

$$g_{21}(s) = -4.4533 \cdot 10^3 s^3 - 3.2461 \cdot 10^6 s^2 -$$

$$- 1.4362 \cdot 10^7 s - 1.2403 \cdot 10^9$$

$$g_{22}(s) = 2.506 \cdot 10^2 s^4 + 1.6229 \cdot 10^4 s^3 +$$

$$+ 6.6134 \cdot 10^6 s^2 + 3.0476 \cdot 10^6 s + 2.4813 \cdot 10^9$$

$$\Delta(s) = s^5 + 1.2308 \cdot 10^2 s^4 + 6.993 \cdot 10^4 s^3 +$$

$$+ 1.5098 \cdot 10^6 s^2 + 3.5504 \cdot 10^8 s + 8.2333 \cdot 10^{-6} \quad (13)$$

Fifth-order elements of the transfer matrix $g_{11}(s)$ and $g_{22}(s)$ were reduced to the third-order using Matlab Toolbox. Reduced elements are given by (14) and they a well represent identified model (13) [6].

$$g_{11}(s)^{\text{Ex.1}} = \frac{191 \cdot s^2 + 593 \cdot s + 75911}{s^3 + 14.3 \cdot s^2 + 5620.5 \cdot s} \quad (14)$$

$$g_{22}(s)^{\text{Ex.1}} = \frac{102 \cdot s^2 - 1041 \cdot s + 39243}{s^3 + 16.8 \cdot s^2 + 5614.8 \cdot s}$$

Appropriate choice of parameters α , β and ξ define the position of the poles in the complex plane. The other coefficients are known from (6). In the all three examples the following values of the parameters are taken $\alpha=12$, $\beta=1$ and $\xi=1$. In this example, according (14) natural frequency is $\omega_n=7.15$ rad/s (for $g_{11}^{\text{Ex.1}}$) and $\omega_n=8.4$ rad/s (for $g_{22}^{\text{Ex.1}}$). Controller parameters calculated from (12) are:

$$K_{p1}=0.4866 ; K_{i1}=0.4131 ; K_{p2}=1.0706 ; K_{i2}=1.5224$$

Values for derivative gains are too high and due to that they were not taken into consideration. That is the potential lack of this procedure.

Example 2.

In this example, polynomial coefficients in the (14) are increased for 20 % to obtain (15).

$$g_{11}(s)^{\text{Ex.2}} = \frac{229.2 \cdot s^2 + 711.6 \cdot s + 91093.2}{s^3 + 17.16 \cdot s^2 + 6744.6 \cdot s} \quad (15)$$

$$g_{22}(s)^{\text{Ex.2}} = \frac{122.4 \cdot s^2 - 1249.2 \cdot s + 47091.6}{s^3 + 20.16 \cdot s^2 + 6737.76 \cdot s}$$

According (15) natural frequency is $\omega_n= 8.58$ rad/s (for $g_{11}^{\text{Ex.2}}$) and $\omega_n= 10.08$ rad/s (for $g_{22}^{\text{Ex.2}}$). Afterward, controller gains from (12) are:

$$K_{p1}=0.4866 ; K_{i1}= 0.7139 ; K_{p2}=1.0706 ; K_{i2}= 2.6308$$

Example 3.

Coefficients in the (14) are decreased for 20 % in this case. Now diagonal elements of the (1) are given by (16).

$$g_{11}(s)^{\text{Ex.3}} = \frac{171.9 \cdot s^2 + 533.7 \cdot s + 68319.9}{s^3 + 12.87 \cdot s^2 + 5058.45 \cdot s} \quad (16)$$

$$g_{22}(s)^{\text{Ex.3}} = \frac{91.8 \cdot s^2 - 936.9 \cdot s + 35318.7}{s^3 + 15.12 \cdot s^2 + 5053.32 \cdot s}$$

Here, natural frequency is $\omega_n= 6.435$ rad/s (for $g_{11}^{\text{Ex.3}}$) and $\omega_n= 7.56$ rad/s (for $g_{22}^{\text{Ex.3}}$). From (12) follows:

$$K_{p1}=0.4866 ; K_{i1}= 0.3012 ; K_{p2}=1.0706 ; K_{i2}= 1.1098$$

Based on configuration in Fig.1, proposed decoupling control was simulated using Matlab/Simulink. Simulations have been carried out for the two cases regarding reference functions (signals) r_1 and r_2 . In the first case r_1 is unit sine function and r_2 is unit step function, and vice versa in the second case. System responses are shown in Fig. 3. and 4, respectively. These figures show responses for the four types of controllers in the combination with static inverted decoupler and one response without decoupler that was controlled in [7]. It is noticeable that P controllers give the best reference tracking. This fact cancels aforementioned possible lack of the proposed procedure, because it is important that at least one type of controller can satisfies defined requirements for system dynamic behavior. The most appropriate value for the proportional gain K_p is obtained when non-dominant pole has 12 time higher absolute value of real part than the three dominant poles.

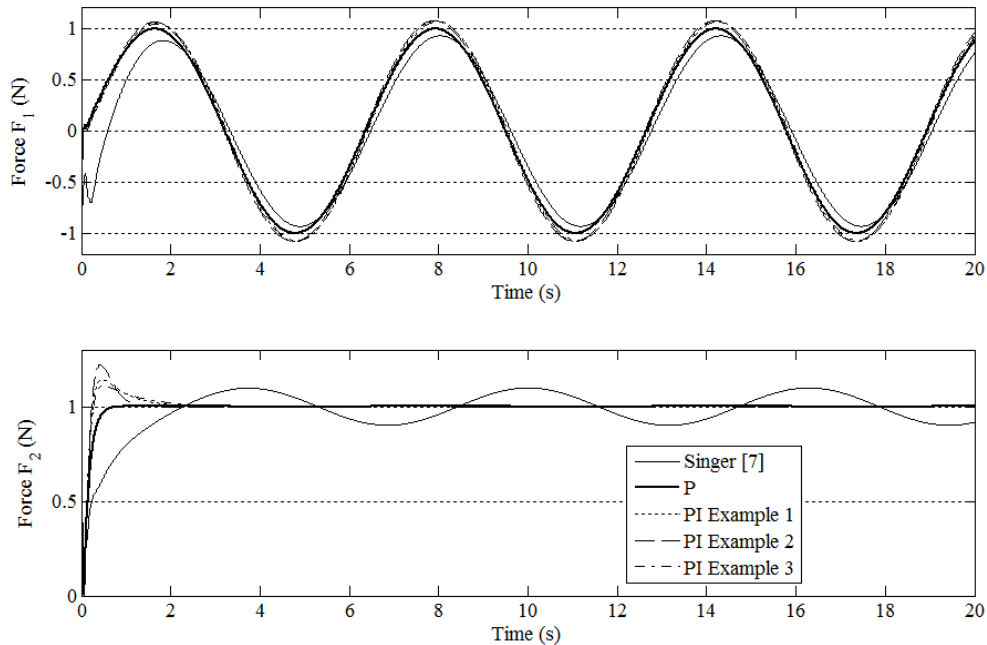


Fig. 3. Forces on the cylinders (r_1 – unit sine function, r_2 – unit step function).

PI controllers gives lower quality of responses. Observing the values of K_p in the explored three examples, it is also noticeable that P controller is the least sensitive to the mod-

el perturbations, i.e. model uncertainties. In comparison with [7] (the case without decoupling) improvement in the compensation of interaction between loops is obvious.

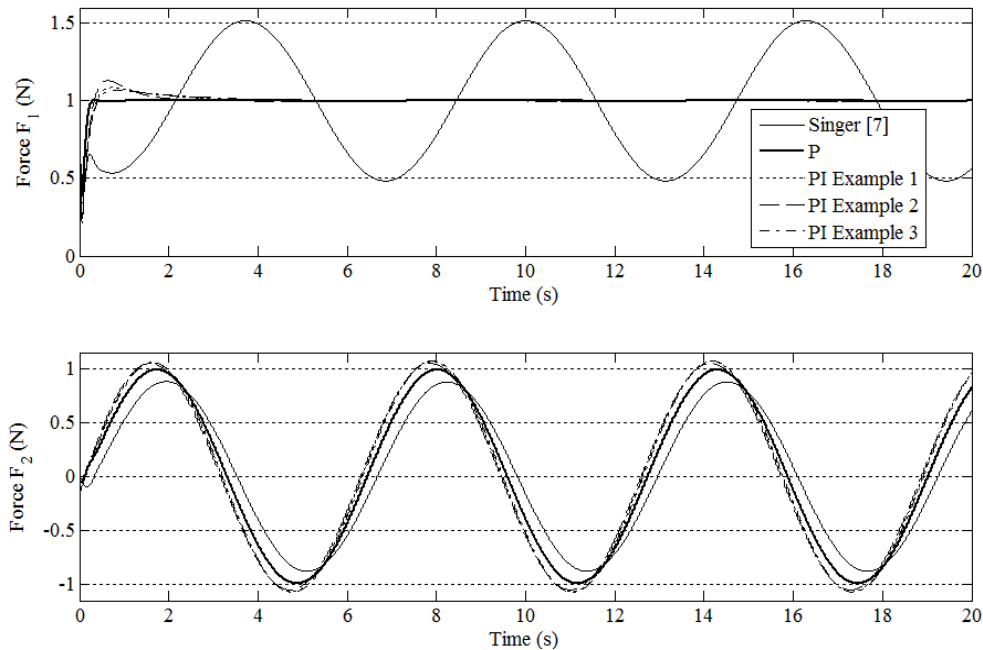


Fig. 4. Forces on the cylinders (r_1 – unit step function, r_2 – unit sine function).

V CONCLUSIONS

Proposed procedure for PID controller design is extension of the dominant pole placement method to the third-order objects with two left half plane zeros. After calculation of controller gains, the most suitable controller type can be chosen. It is proved that difference between absolute values of real part of non-dominant and dominant poles in some control algorithms should be taken more than usually in literature suggested four times. Controllers tuned based on presented approach are compatible with previously decoupled objects. This is confirmed on the TITO electrohydraulic system for structural testing, where P controller in the combination with static inverted decoupler enables good system performances, especially regarding reference tracking and cancelation of mutual coupling.

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