Design of PID Controllers for the System of a Pump Controlled Hydro-Motor

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Abstract: *This paper presents design of a PID controller for the system of a pump-controlled motor with a long transmission line. By using possibilities offered by computers and software the graphical method is used in the design of PID controllers. The D–decomposition method including system performances, damping and settling time was applied. This type of controllers frequently satisfies practical needs because the feedback loop system is fast, and the static error is reduced to 0. The proposed method allows easy design of PID controllers in significant modifications of lengths of the transmission line. The system performances are included in a new manner without the need for calculation of Chebyshev functions for every change of the damping coefficient thanks to strong software support.*

Keywords: PID controller, D-decomposition, relative stability, settling time, robustness

1. INTRODUCTION

Increasingly strict and wide requirements regarding dis-placement hydrostatic power transmitters have recently appeared in the sense of simultaneous accomplishment of high power exploitation degrees, high speed of response with the reduction of price [1-4]. This particularly refers to high power systems and systems with variable load (build-ing and mining machines, agricultural machines, transpor-tation machines, machine tools, etc). It is obvious that these requirements result in the need for more intense de-velopment of systems with displacement control in relation to the systems with damping control. It is obvious that these requirements result in the need for more intense de-velopment of systems with displacement control in relation to the systems with damping control. One of the main preconditions for quality and reliable operation of a high power system is the stable and quality operation of the system for automatic control of hydrostatic power trans-mitter, the pump-controlled motor with long transmission lines (Figure 1). The existence of a long transmission line in this system makes its dynamics rather complex because the physical values, pressure and flow, which characterize the transfer of energy along the long transmission line de-pend both on the time coordinate and the space coordinate. Dependence of these physical values on the space coordinate, too, conditions that in the mathematical description of the long transmission line the space distribu-tion cannot be neglected, so that it is described by a model with distributed parameters. Models with distributed pa-rameters are described by partial differential equations and the model obtained is of an infinitesimally high order [5-9]. In addition to mathematical modeling of the long trans-mission line by means of the model with distributed pa-rameters, it is possible to describe the long transmission line by common differential equations, i.e. models with lumped parameters [1-4] because solving common differential equations makes considerably fewer difficulties in compar-ison with solving partial differential equations. The authors of this paper considered the problem of modelling and dynamic behaviour of such systems in a very systematic way, and the results are presented in several papers, the most significant of which are [4], [10] and [12]. Reference [10] gives a complete mathematical model of the system of a pump-controlled motor with a long transmission line by means of a model with lumped parameters where the long transmission line is divided into n equal “Π” segments. The mathematical model thus obtained is of high order but by applying the appropriate methodology its order is re-duced, which considerably increases its use value. From the aspect of control, reference [10] presents design of a P controller by applying the Nyquist criterion, including system performances, damping and settling time. The P controller designed in this way, for the described system, eliminates the occurrence of oscillations of the transfer characteristic due to the existence of the long transmission line. However, design of the P controller could not eliminate the error, so that a PI controller was designed in order to solve that problem.

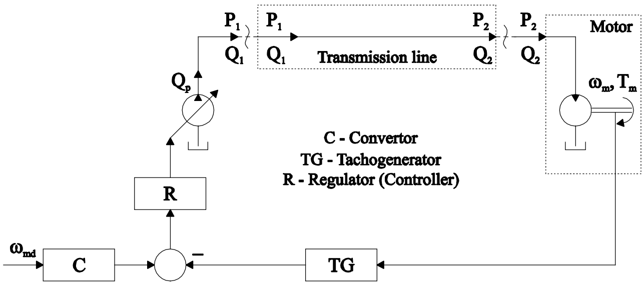

Figure 1: Symbolic diagram of the closed automatic control system of a pump-controlled motor with a long transmission line

This paper proposes the procedure for design of the PID controller for systems of a pump-controlled motor.

The starting point is a requirement which establishes a direct relation between the IE criterion and the integral gain (the higher the integral gain, the smaller the value of the IE criterion). The result is extended by introducing engineering specifications (settling time and relative stability). It results in a simple and efficient procedure for design of the PID controller for systems of a pump-controlled motor.

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2. DESIGN PROCEDURE OF PID CONTROLLER

Since the problem of disturbance load rejection is reduced to the minimization of IE criteria, it is also considered in this paper, but engineering constraints are introduced on:

i) relative stability
ii) settling time

This is the basis for development of a simple graphical method based on D-decomposition [11-16].
The transfer function of the PID controller is:

$$W_R = K_p + \frac{K_i}{s} + K_d \cdot s \tag{1}$$

$$W_R = K_p(1 + \frac{1}{T_i s} + T_d \cdot s) \tag{2}$$

Based on (1) and (2), we can get:

$$K_i = \frac{K_p}{T_i}, K_d = K_p T_d \tag{3}$$

$$T_i = nT_d \tag{4}$$

Based on equations (3) i (4), the relation between K_d and K_i is obtained in the following form:

$$K_d = \frac{K_p^2}{n \cdot K_i} \tag{5}$$

The transfer function of the process is represented in the form:

$$W_P(s) = \frac{N(s)}{M(s)} = \frac{\sum_{k=0}^{m} b_k s^k}{\sum_{k=0}^{n} a_k s^k}, \quad m \le n \tag{6}$$

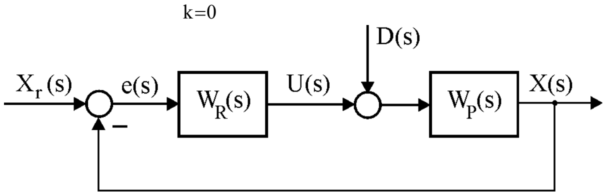

Figure 1: *Automatic control system*

The characteristic equation of the automatic control system from Fig1 is determined by the equation:

$$f(s) = 1 + W_R(s)W_P(s) = 0 \tag{7}$$

Putting K_d = 0 in the equation (1), the problem is reduced to PI controller design.

$$f(s) = 1 + (K_p + \frac{K_i}{s}) \cdot \frac{N(s)}{M(s)} = 0 \tag{8}$$

$$f(s) = s \cdot M(s) + (K_p s + K_i) \cdot N(s) = 0 \tag{9}$$

$$f_1(s) = s \cdot M(s) = \sum_{k=0}^{n} a_k s^{k+1} \tag{10}$$

By connecting (9) and (10), the final expression for the characteristic equation of the automatic control system in the complex domain is obtained as follows:

$$f(s) = f_1(s) + (K_p s + K_i) \cdot N(s) = 0 \tag{11}$$

Taking into account (11), it is necessary to express the complex number *s* in a suitable form and use it for establishing the relation between the damping degree ξ and the variable parameters of the controller K_p and K_i contained in the characteristic equation (11) for the automatic control system. This is how the area from the “*s*” plane below the straight line ξ=const. (Fig. 2), is mapped in the area of the corresponding damping coefficient represented by the curve ξ=const., in the parameter plane of tuning parameters of the controller (K_p, K_i) with the condition for observation of the integral gain K_i as a parameter that fulfills the condition of minimum of IE criteria for the corresponding level of the damping coefficient ξ.

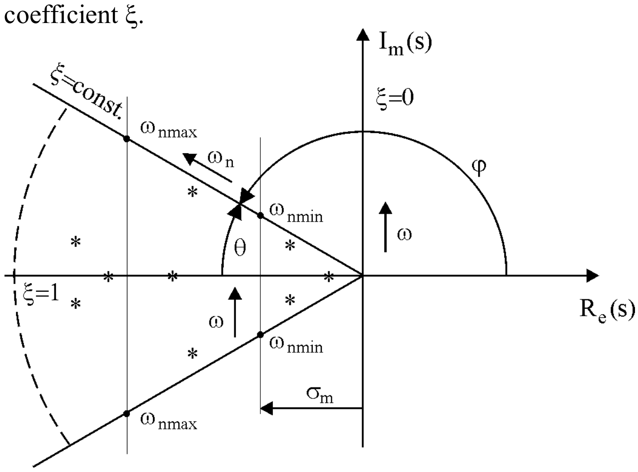

Figure 2: Area with the required settling time and relative stability

Since:

$$s = -\omega_n \xi + j\omega_n \sqrt{1-\xi^2} \tag{12}$$

By connecting (11) with (12) the characteristic equation of the automatic control system obtains the form:

$$f_1(\xi, \omega_n) + \left[K_p(-\xi\omega_n + j\omega_n\sqrt{1-\xi^2}) + K_i \right] \cdot N(\xi, \omega_n) = 0 \tag{13}$$

$$f_1(\xi, \omega_n) = \alpha(\xi, \omega_n) + j\beta(\xi, \omega_n) \tag{14}$$

where $\alpha(\xi,\omega_n)$ and $\beta(\xi,\omega_n)$ represent the real and imaginary parts of the polynomial $f_1(\xi,\omega_n)$.

$$\alpha(\xi, \omega_n) = \sum_{k=1}^{n} a_{k-1}(-1)^k \omega_n^k T_k(\xi) \tag{15}$$

$$\beta(\xi, \omega_n) = \sqrt{1-\xi^2} \sum_{k=1}^{n} a_{k-1}(-1)^{k+1} \omega_n^k U_k(\xi) \tag{16}$$

$$\beta(\xi, \omega_n) = \sqrt{1-\xi^2} B(\xi, \omega_n) \tag{17}$$

where T_k and U_k are Chebyshev functions of the first and second types for which the following recurrent equations hold:

$$T_{k+1} = 2\xi T_k - T_{k-1}, U_{k+1} = 2\xi U_k - U_{k-1} \tag{18}$$

$$T_0 = 1, T_1 = \xi, U_0 = 0, U_1 = 1. \tag{19}$$

$$N(\xi, \omega_n) = \gamma(\xi, \omega_n) + j\delta(\xi, \omega_n) \tag{20}$$

where $\gamma(\xi,\omega_n)$ and $\delta(\xi,\omega_n)$ represent the real and imaginary parts of the polynomial $N(\xi,\omega_n)$ and they are determined based on the following equations:

$$\gamma(\xi, \omega_n) = \sum_{k=0}^{m} b_k(-1)^k \omega_n^k T_k(\xi) \tag{21}$$

$$\delta(\xi, \omega_n) = \sqrt{1-\xi^2} \sum_{k=0}^{m} b_k(-1)^{k+1} \omega_n^k U_k(\xi) \tag{22}$$

$$\delta(\xi, \omega_n) = \sqrt{1-\xi^2} D(\xi, \omega_n) \tag{23}$$

By connecting the equation (13) with equations from (14) to (23), after appropriate mathematical transformations and separating the real and imaginary parts, the following system of equations is obtained:

$$\begin{aligned} K_p\left[\xi\cdot\omega_n\cdot\gamma(\xi,\omega_n)+\sqrt{1-\xi^2}\cdot\omega_n\cdot\delta(\xi,\omega_n)\right]-K_i\cdot\gamma(\xi,\omega_n)=\alpha(\xi,\omega_n)\\ K_p\left[\xi\cdot\omega_n\cdot\delta(\xi,\omega_n)-\sqrt{1-\xi^2}\cdot\omega_n\cdot\gamma(\xi,\omega_n)\right]-K_i\cdot\delta(\xi,\omega_n)=\beta(\xi,\omega_n)\end{aligned} \tag{24}$$

By solving the system of equations at $\omega_n \neq 0,\quad 0\leq\xi<1$, the expressions for the parameters Kp and Ki of the PI controller are obtained.

$$K_i=\omega_n\left[\xi+\sqrt{1-\xi^2}\cdot\frac{\beta(\xi,\omega_n)\cdot\delta(\xi,\omega_n)+\alpha(\xi,\omega_n)\cdot\gamma(\xi,\omega_n)}{\beta(\xi,\omega_n)\cdot\gamma(\xi,\omega_n)-\alpha(\xi,\omega_n)\cdot\delta(\xi,\omega_n)}\right]\cdot K_p \tag{25}$$

3. CONTROL OF A PROCESS WITH A LONG TRANSMISSION LINE

In order to show the efficiency of the proposed method for PID controller design we have performed simulations in MATLAB program for transfer function of the process $W_P(s)$:

$$W_P(s)=\frac{1}{5.2\cdot10^{-25}s^{10}+9.23\cdot10^{-22}s^9+9.677\cdot10^{-19}s^8+7.838\cdot10^{-16}s^7+4.592\cdot10^{-13}s^6+2.072\cdot10^{-10}s^5+7.257\cdot10^{-8}s^4+\ldots}$$
$$\ldots+1.755\cdot10^{-5}s^3+2.962\cdot10^{-3}s^2+0.243s+1.248 \tag{26}$$

The transfer function described by (26), represents a mathematical model of a pump controlled hydromotor, where the variable flow pump and the hydromotor of constant flow are connected by means of a long transmission line [17].

Based on the programme created in MATLAB, according to the proposed procedure, the parameters of the PI controller can be determined for the transfer function of the process described by (26), so that the closed loop of the system could possess the required damping coefficient ($\xi\geq 0.6$).

Figure 3 shows the parameter plane (K_p-K_i) for values of damping coefficient ξ= 0.6. From the boundary curve, it can be selected the maximum value of the integral gain and the corresponding value of the proportional gain. This value of integral gain enables a minimum of IE criteria. The area under the curve represents the area with the damping coefficient $\xi\geq 0.6$.

On figure 5 it is shown the comparative response of the system between PI and PID controller. PID controller is designed in such a way that the value of the gain K_d is determined based on equation (5). In this, care should be taken to be satisfied the condition n≥4 [18-20]. In this determination of differential gain K_d, it was taken into account that the system constantly maintains the value of the damping coefficient $\xi\geq 0.6$, while at the same time the minimum IE criteria is satisfied. From Figure 5, it can be shown that increasing the number n from 4 to 10 in the designed PID controller, leads to significant reduction in the settling time from 216 ms to 130 ms. Also compared to the PI regulator we can see that there has been a reduced jump from 43% to 28%. An overview of all these performances is given in detail in Table 1. Figure 6 shows how the designed PID controller for n = 10 suppresses the load disturbance in relation to the PI controller.

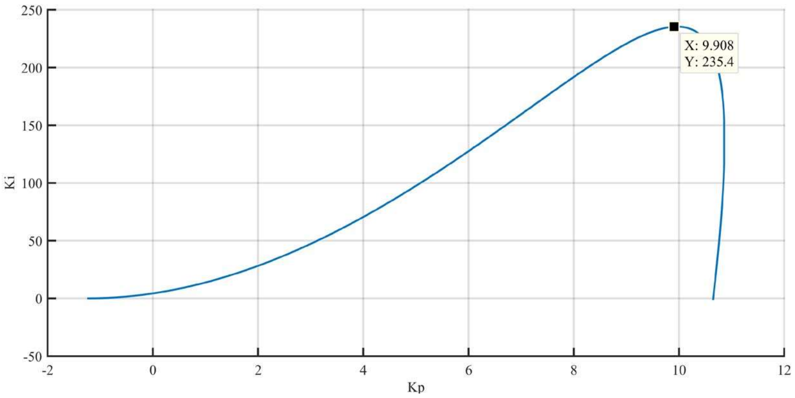

Figure 3: Parameter plane for damping coefficient ξ= 0.6

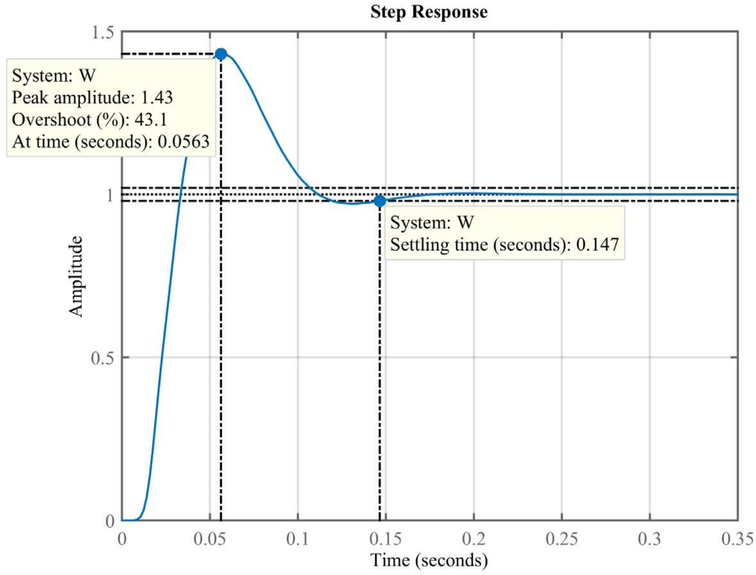

Figure 4: System response regulated by PI controller for *ξ= 0.6*

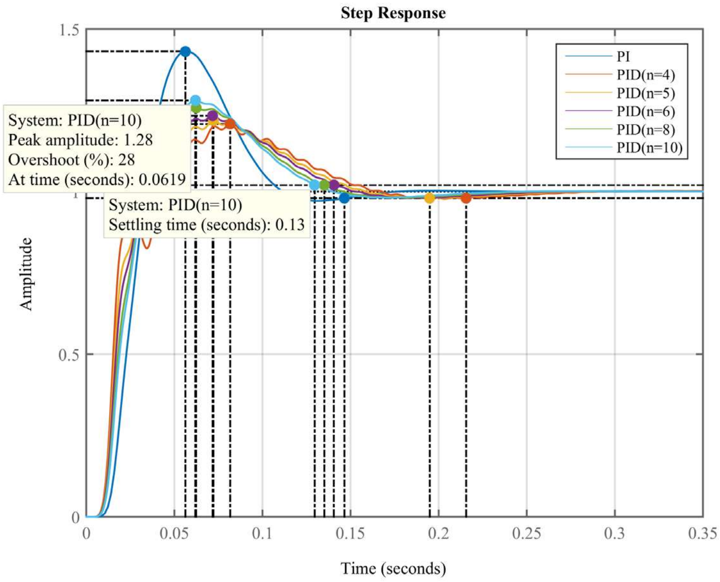

Figure 5: Comparative response of a system regulated by PI and PID controller for different value of number n

Table 1: Comparative presentation of the results of design of the PI controller with PID controller

Method	K_p	K_i	K_d	Overshoot (%)	Settling time t_s (ms)	Phase margin φ_m (degrees)
PI	9.908	235.4	-	43.1	147	35.7
PID (n=4)	9.908	235.4	0.1043	**20.8**	216	60.7
PID (n=5)	9.908	235.4	0.0834	**21.9**	**195**	55.7
PID (n=6)	9.908	235.4	0.0695	**23.3**	**140**	52.5
PID (n=8)	9.908	235.4	0.0521	**25.7**	**135**	48.4
PID (n=10)	9.908	235.4	0.0417	**28**	**130**	45.9

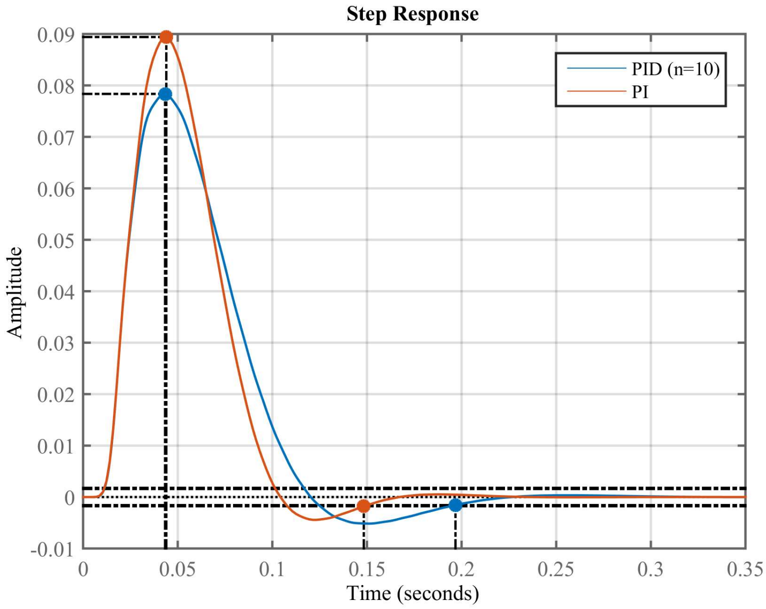

Figure 6: Comparative response of a system regulated by PI and PID controller in relation to suppression of load disturbance

4. CONCLUSION

Based on everything said, we can conclude that it has been developed an efficient and simple graphical methods for design of the PID controller, which achieves high performances for a broad range of linear processes. The process of high order has been considered, in which the variable flow pump controls the hydromotor, where a connection between the pump and the hydromotor is realized by a long transmission line. In comparison with the procedures for tuning of the PID controller proposed in literature, the method described in this paper is characterized by great simplicity and clear engineering specifications. The results of simulations show good robustness in relation to unmodelled dynamics as well as superiority over some other methods of tuning of controllers. The proposed method is suitable for on line real-time realization and for auto tuning of the PID controller.

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