About Eigen-Problem of Single Photon Hamiltonian

N.V. Delić, S. Pelemiš, J.P. Šetrajčić

Abstract - Altered state of consciousness is a consequence of quantum change which is presented by photons. There is not exist adequate theory, to describe this phenomenon. Linearized single photon Hamiltonian is used for the analysis of its features in coordinate systems. It can be concluded that photon behavior is equally determined by its translation characteristics (impulse components) and its rotation characteristics expressed by way of spin operators.

I. LINEARIZED PHOTON HAMILTONIAN

Electroencephalography (EEG) is a study of changing electrical potential of the brain. According to their frequency brainwaves are divided into 4 main groups, also referred to as "brain states". Altered state [1] of consciousness is a consequence of quantum change which is presented by photons. There is not exist adequate theory, to describe this phenomenon.

Linearized single photon Hamiltonian is used for the analysis of its features in coordinate systems of various geometries. As it could have been expected, based on the general theory of relativity, it turned out that space geometry and physical features are closely interrelated.

Classical expression for free photon energy is:

$$E = c\sqrt{p_x^2 + p_y^2 + p_z^2} \tag{1}$$

where *c* is the light velocity in vacuum and p_x, p_y, p_z are the components of photon momentum.

If instead of classical momentum components we use quantum-mechanical operators $p_\nu \to \hat{p}_\nu = -i\hbar \dfrac{\partial}{\partial x_\nu}$; $\nu = x,y,z$; where $\hbar = \dfrac{h}{2\pi} =$ 1,05456·10^{-34} Js is Dirac constant, we obtain quantum-mechanical single photon Hamiltonian:

$$\hat{H} = \pm c\sqrt{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2} \tag{2}$$

N.V. Delić is with the Primary School "Svetozar Marković – Toza", Novi Sad, Vojvodina – Serbia, E-mail: bora@im.ns.ac.yu

S. Pelemiš is with the Faculty of Technology Zvornik, University of East Sarajevo, Karakaj bb, Zvornik, Republic of Srpska – Bosnia and Herzegovina, E-mail: alannica@nspoint.net

J.P. Šetrajčić is with the Department of Physics, Faculty of Sciences, University of Novi Sad, Trg D. Obradovića 4, 21000 Novi Sad, Vojvodina – Serbia, E-mail: bora@if.ns.ac.yu

Hamiltonian (2) is not a linear operator which contradicts the principle of superposition [2,3]. Klein and Gordon skirted this problem solving the eigen problem of square of Hamiltonian (2):

$$\hat{H}^2\varphi = E^2\varphi \tag{3}$$

since the square of Hamiltonian is a linear operator. This approach has given satisfactory description of single photon behaving. Up to now it is considered that this approach gives real picture of photon. Here will be demonstrated that Klein–Gordon picture of photon is incomplete.

Here we shall try to examine single photon behavior by means of linearized Hamiltonian (2). Linearization procedure is analogous to the procedure which was used by Dirac in the analysis of relativistic electron Hamiltonian.

We shall take that

$$\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2 = \left(\hat{\alpha}\hat{p}_x + \hat{\beta}\hat{p}_y + \hat{\chi}\hat{p}_z\right)^2 \tag{4}$$

i.e. we shall transform the sum of squares into the square of the sum using $\hat{\alpha}, \hat{\beta}$ and $\hat{\chi}$ matrices. In accordance with (4) these matrices fulfill the following relations:

$$\begin{aligned} &\hat{\alpha}^2 = \hat{\beta}^2 = \hat{\chi}^2 = 1 \\ &\hat{\alpha}\hat{\beta} + \hat{\beta}\hat{\alpha} = \hat{\alpha}\hat{\chi} + \hat{\chi}\hat{\alpha} = \hat{\beta}\hat{\chi} + \hat{\chi}\hat{\beta} = 0 \end{aligned} \tag{5}$$

It is easy to show that (5) conditions are fulfilled by Pauli's matrices

$$\hat{\alpha} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\beta} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\chi} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

Combining (6), (4) and Hamiltonian (2), we obtain linearized photon Hamiltonian which completely reproduces the quantum nature of light [2]. The form of linearized single photon Hamiltonian is:

$$\begin{aligned} \hat{H} &= \pm c\begin{pmatrix} \hat{p}_z & \hat{p}_x - i\hat{p}_y \\ \hat{p}_x + i\hat{p}_y & -\hat{p}_z \end{pmatrix} = \\ &= \pm \frac{\hbar c}{i}\begin{pmatrix} \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial x} - i\dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial x} + i\dfrac{\partial}{\partial y} & -\dfrac{\partial}{\partial z} \end{pmatrix} \end{aligned} \tag{7}$$

978-1-4244-1882-4/08/$25.00 © 2008 IEEE

Since linearized Hamiltonian is a 2×2 matrix, photon eigen states must be columns and rows with two components. Operators of other physical quantities must be represented in the form of diagonal 2×2 matrices.

At the end of this presentation, it is important to underline the orbital moment operator $\begin{pmatrix} \hat{L} & 0 \\ 0 & \hat{L} \end{pmatrix}$; $\hat{\vec{L}} = \hat{\vec{r}} \times \hat{\vec{p}}$ does not commute with Hamiltonian (7). It means that it is not integral of motion as in Klein-Gordon's theory. It can be shown that integral of motion represents total momentum $\begin{pmatrix} \hat{\vec{J}} & 0 \\ 0 & \hat{\vec{J}} \end{pmatrix}$; where $\hat{\vec{J}}$ is the sum of orbital moment $\hat{\vec{L}}$ and rotation momentum $\hat{\vec{S}}$ which corresponds to $\frac{1}{2}$ spin.

II. Brain Photon Energy Spectrum

Brain photons will be analyzed using linearized photon Hamiltonian. It is known that a photon Hamiltonian:

$$\hat{H} = \pm c\sqrt{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2} \tag{8}$$

where c is the speed of light and $\hat{p}_x, \hat{p}_y$ and $\hat{p}_z$ are impulse components is not a linear operator and as such cannot be applied because of superposition principle.

Representing the sum of squares before the root sign in the form of square of the sum, Hamiltonian (8) is reduced to the linear form:

$$\hat{H} = c(\hat{\alpha}\hat{p}_x + \hat{\beta}\hat{p}_y + \hat{\chi}\hat{p}_z) \tag{9}$$

In the form (9), $\hat{\alpha}, \hat{\beta}, \hat{\chi}$ are 2-by-2 Pauli matrices which fulfill the following commutation relations:

$$\begin{aligned} \left[\hat{\alpha}, \hat{\beta}\right] &= 2i\hat{\chi} \\ \left[\hat{\chi}, \hat{\alpha}\right] &= 2i\hat{\beta} \\ \left[\hat{\beta}, \hat{\chi}\right] &= 2i\hat{\alpha} \end{aligned} \tag{10}$$

Based on the fact that the components of spin operator fulfill the commutation relations :

$$\begin{aligned} \left[\hat{S}_x, \hat{S}_y\right] &= i\hbar\hat{S}_z \\ \left[\hat{S}_z, \hat{S}_x\right] &= i\hbar\hat{S}_y \\ \left[\hat{S}_y, \hat{S}_z\right] &= i\hbar\hat{S}_x \end{aligned} \tag{11}$$

comparing (10) and (11), we come to the following correspondence between spin operators and Pauli matrices:

$$\begin{aligned} \hat{\alpha} &= \frac{2}{\hbar}\hat{S}_x \\ \hat{\beta} &= \frac{2}{\hbar}\hat{S}_y \\ \hat{\chi} &= \frac{2}{\hbar}\hat{S}_z \end{aligned} \tag{12}$$

After substitution of (12) in (9) linearized photon Hamiltonian is obtained in the form of the product of components of photon impulse and corresponding spin components:

$$\hat{H} = \frac{2c}{\hbar}(\hat{p}_x\hat{S}_x + \hat{p}_y\hat{S}_y + \hat{p}_z\hat{S}_z) \tag{13}$$

III. Conclusion

Based on (13), it can be concluded that photon behavior is equally determined by its translation characteristics (impulse components) and its rotation characteristics expressed by way of spin operators.

Acknowledgements

The authors are very gratefull to Prof. Dr. Bratislav S. Tošić for usefull sugestions.

This work (JPŠ) was partially supported by the Serbian Ministry of Science and Technology: Grant No 141044.

References

[1] N.V.Delić, J.P.Šetrajčić, A.S.Utješanović, V.M.Zorić, D.Lj.Mirjanić, S.K.Jaćimovski, Energy Aspect of Consciousness Layers, *Proceedings 1st Annual ECPD International Symposium on Scientific Bases of Consciousness*, 321-326 (1997).

[2] K.Gottifried, Quantum Mechanics: Fundamentals, p.412, Addison–Wesley, Reading, Mass, 1989. (reprint of 1966 Benjamin edition).

[3] A.M.Kadin, Quantum Mechanics without Complex Numbers:A simple Model for the electron Wavefunction Including Spin, ArXiv Quantum Physics preprint, available online at http://arxiv.org/abs/quant-ph/0502139, 2005.

[4] M.Sapaznjikov, Antimir-Realnosti, p.33,34, Znanie, Moscow, 1983 (in Russian).