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The influence of different lamina positions on buckling properties of composites plates under biaxial compression

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Abstract. The present work focused on the buckling of composite plates subjected to biaxial compression using classical laminated plate theory (CLPT). The two composite plates are bonded by an elastic medium. Classical laminated plate theory (CLPT) is utilized for deriving the governing equations. Difference between 4-ply symmetric lamination scheme with different material properties and fiber orientation angle (0° and 30°) is shown. The study show that the mechanical properties of the composite materials depending on the fiber orientation angle and lamina positions.

1. Introduction

Composite materials are materials made from two or more constituent materials with specific different physical or chemical properties that when combined produce a material with characteristics different from the individual components. Generally, a composite material is composed of reinforcement (fibres, particles, flakes, and/or fillers) embedded in a matrix (polymers, metals, or ceramics). The matrix holds the reinforcement to form the desired shape while the reinforcement improves the overall mechanical properties of the matrix. Currently, the analysis of the behavior of the laminated plates is an active research area because of their complex behavior [1]. The majority of the research work carried out on the laminated plates, is devoted to the stress determination, deformations or displacements of flexional origin. The structural instability becomes an important concern in a reliable design of composite plates. Several studies on laminated plates stability were concentrated on rectangular plates [2-5]. It is known that buckling strength of the rectangular plates depends on the boundary conditions [3], plies orientation [3,4,6] and geometrical ratio [3,5-7]. The thin composites structures which are largely used become unstable when they are subjected to mechanical or thermal loadings which leads to buckling. Consequently, the buckling behavior is a significant factor of the sure and reliable design [8]. To predict buckling load and deformation mode of a structure, the linear analysis can be used as an evaluation technique [9]. Generally, the analysis of the laminated plates is more complicated than the analysis of an isotropic and homogeneous material [10]. Finite element method is used for the analysis of the buckling behavior of the notched antisymmetrical fibers plates under compression [11].

The main contribution of this work is to perform a composite laminated plates analysis by using the Classical Laminated Plate Theory (CLPT) is described in [12-14]. Various geometries of the plates subjected to compressive load are studied.



2. Governing equations of biaxially compressed composites plates

Let us consider composite plate the length of a , width b and height h , as shown in figure 1.

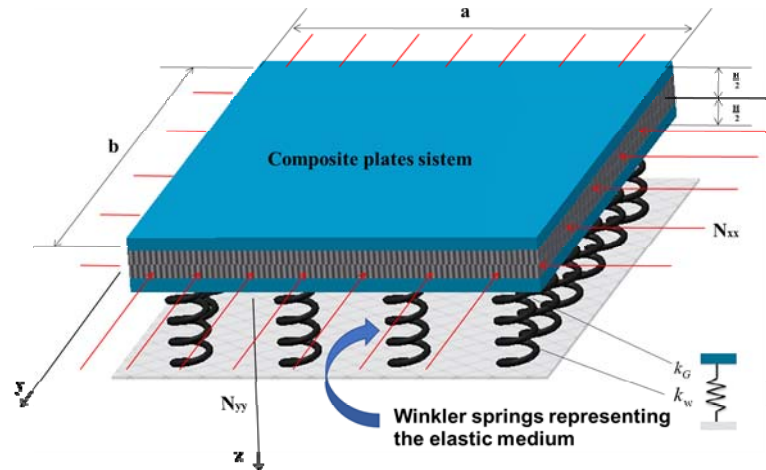


Figure 1. Schematic diagram of composite plates system subjected to biaxial compression.

Composite plates are assumed to be coupled by an elastic medium and subjected to biaxially compression. The composite plates are surrounded by external elastic medium. The external medium is modeled as Pasternak-type foundation which is equivalent to Winkler modulus parameter k_w and shear modulus parameter k_G of polymer matrix. Bending displacements of the plate-1 and plate-2 are $w_1(x, y, t)$ and $w_2(x, y, t)$, respectively. It was assumed that each composite plate had the length, a and width, b . We assume that composite plates are biaxially compressed by forces N_{xx} and N_{yy} in the directions of x and y axes, respectively (figure 1).

The governing equation for biaxially compressed orthotropic composite plate embedded in an elastic medium [15], which is based on Classical Laminated Plate Theory CLPT, have following form

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + k_w w - k_G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad (1)$$

We assume that composite plate is biaxially compressed in the directions of x and y axes, $N_x = N_y$. Now we can define compression ratio which equals the ratio between the forces acting in y and x directions

$$\delta = \frac{N_{yy}}{N_{xx}} \rightarrow N_{yy} = \delta N_{xx} \quad (2)$$

Substitution of equation (2) in equation (1) we derive the general form of governing equation

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + N_x \left(\frac{\partial^2 w}{\partial x^2} + \delta \frac{\partial^2 w}{\partial y^2} \right) + k_w w - k_G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad (3)$$

Before solving constituent equation (3), boundary conditions should be defined. In this study it is assumed that all edges on both nanoplates are simply supported. This means that both the displacements and moments at the composite plates edges are zero. This can be expressed by following equations

$$w_i(0, y, t) = 0, w_i(a, y, t) = 0, w_i(x, 0, t) = 0, w_i(x, b, t) = 0 \quad i = 1, 2 \quad (4)$$

$$M_i(0, y, t) = 0, M_i(a, y, t) = 0, M_i(x, 0, t) = 0, M_i(x, b, t) = 0 \quad (5)$$

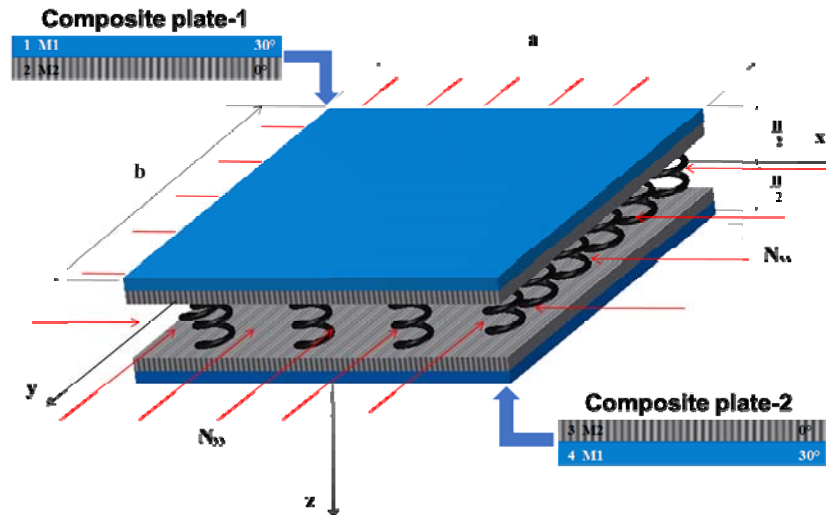


Figure 2. Composite plate system coupled by an elastic medium.

If in the middle of the four-layer laminate system we insert an elastic medium that separate the laminate into two symmetric parts, we will have two composite plates with two laminae (figure 2) whose main equations are:

Plate 1

$$D_{11} \frac{\partial^4 w_1}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w_1}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w_1}{\partial y^4} + N_x \frac{\partial^2 w_1}{\partial x^2} + N_y \frac{\partial^2 w_1}{\partial y^2} + k_w (w_1 - w_2) - k_G \nabla^2 (w_1 - w_2) = 0 \quad (6)$$

Plate 2

$$D_{11} \frac{\partial^4 w_2}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w_2}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w_2}{\partial y^4} + N_x \frac{\partial^2 w_2}{\partial x^2} + N_y \frac{\partial^2 w_2}{\partial y^2} + k_w (w_2 - w_1) - k_G \nabla^2 (w_2 - w_1) = 0 \quad (7)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (8)$$

3. Buckling loads of biaxially compressed composite plates

In this section different explicit cases of biaxial buckling will be considered. The composite plate system is subjected to both biaxial as well as biaxial compressive forces. The cases studied will be composite plates buckling with out-of-phase (asynchronous); in-phase (synchronous); and when one of the composite plates is considered to be fixed.

3.1. Asynchronous-type bi-axial buckling (out-of-phase)

Composite plates system is assumed to be bi-axially buckled. Figure 3 shows the three-dimensional configuration of double composite plates system with the asynchronous (out-of-phase) sequence of buckling:

$$w_1(x, y, t) - w_2(x, y, t) \neq 0$$

In out-of-phase, sequence of buckling the nanoplates is buckled in opposite directions. We evaluate the buckling load for the out-of-phase (asynchronous) type buckling and use equations (6,7) for the biaxial buckling solution of double composite plates system.

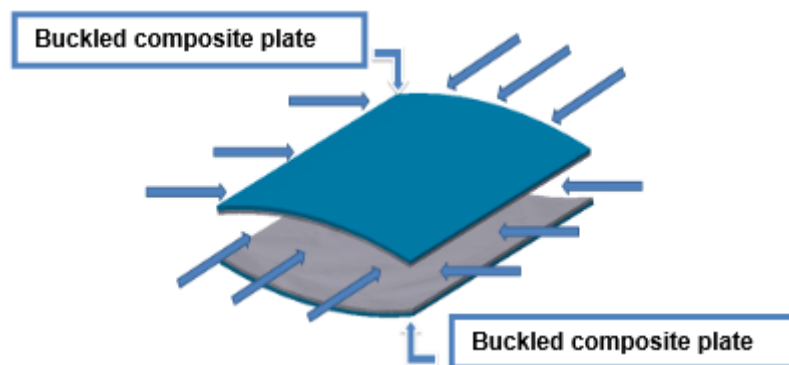


Figure 3. Asynchronous-type bi-axial buckling.

Subtracting equation (6) from equation (7) we get:

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2k_w w - 2k_G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad (9)$$

$$w = w_1 - w_2 - w_2 + w_1 = 2w_1 - 2w_2 = 2(w_1 - w_2) = 2w \quad (10)$$

We assume that the buckling mode of the double-nanoplate system as

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin(\alpha x) \sin(\beta y) \quad (11)$$

In the upper equation:

$$\alpha = \frac{m\pi}{a}, \quad (12)$$

$$\beta = \frac{n\pi}{b}$$

where m and n are the half wave numbers.

Substituting equation (11) into equation (9), we get critical buckling load for asynchronous type of buckling

$$N_{cr} = \frac{D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 + 2k_w + 2k_G(\alpha^2 + \beta^2)}{(\alpha^2 + \delta\beta^2)} \quad (13)$$

3.2. Synchronous-type bi-axial buckling (in-phase)

In-phase (synchronous) sequence type of buckling of the double composite plates under bi-axial compression is also being considered here. The schematic illustration buckling of the orthotropic composite plates is shown in figure 4, which is the first mode synchronous type buckling. In synchronous buckling both the composite plates buckle in the same direction. For the present system, the relative displacements between the two composite plates are

$$w_1(x, y, t) - w_2(x, y, t) = 0$$

In synchronous buckling state, the double composite plates system can be considered to be as one of the composite plates.

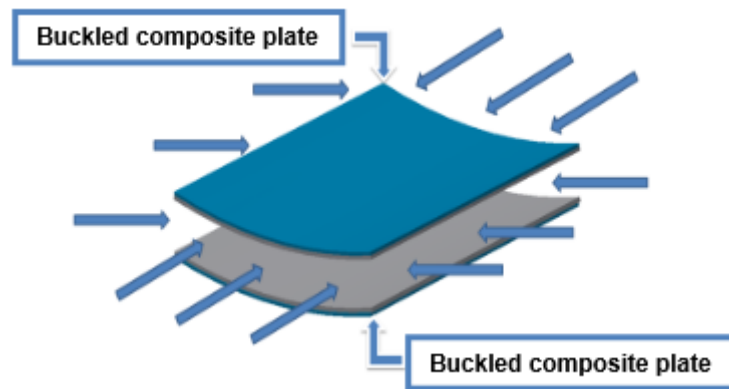


Figure 4. Synchronous-type bi-axial buckling.

We apply the same procedure as earlier for solving equations (6) and (7). For this case of buckling governing equation can be written as

$$D_{11} \frac{\partial^4 w_2}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w_2}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w_2}{\partial y^4} + N_x \frac{\partial^2 w_2}{\partial x^2} + N_y \frac{\partial^2 w_2}{\partial y^2} = 0 \quad (14)$$

$$w_1 = w_2 \quad (15)$$

$$w = w_1 - w_2 = 0$$

Following procedure similar to that of out-of-phase buckling, critical buckling load for synchronous type of buckling can be written as

$$N_{cr} = \frac{D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4}{(\alpha^2 + \delta\beta^2)} \quad (16)$$

For this case the critical buckling load is independent of the stiffness of the coupling springs and biaxial compression of double composite plates system can be effectively treated as a single composite plate.

3.3. Bi-axial buckling with one composite plate fixed

Consider the case of composite plates system when one of the two composite plates is stationary

$$w_2 = 0$$

which is shown in figure 5. For this case of buckling governing equation can be written as

$$D_{11} \frac{\partial^4 w_1}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w_1}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w_1}{\partial y^4} + N_x \frac{\partial^2 w_1}{\partial x^2} + N_y \frac{\partial^2 w_1}{\partial y^2} + k_w w_1 - k_G \left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right) = 0 \quad (17)$$

and critical buckling load for this type of buckling can be written as

$$N_{cr} = \frac{D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 + k_w + k_G(\alpha^2 + \beta^2)}{(\alpha^2 + \beta^2)} \quad (18)$$

In fact, when one of the composite plates in composite system is fixed ($w_2 = 0$), the composite system behaves as composite plate on an elastic medium.

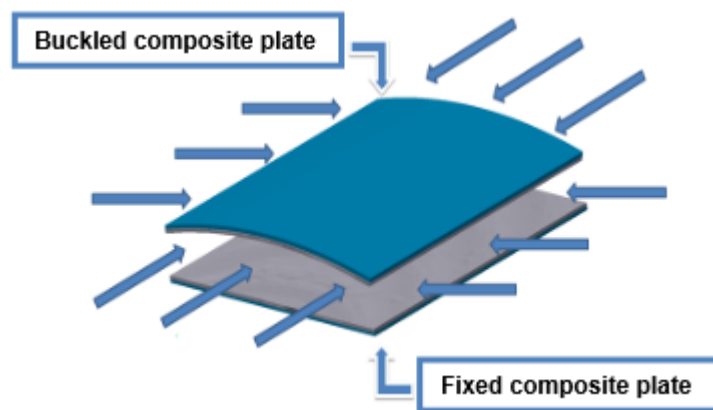


Figure 5. Bi-axial buckling with one composite plate fixed.

4. Numerical results and discussion

In this section follows analysis of four-layer symmetric laminate made of two types of material with different orientation of the fibers. Two types of materials are combined:

- Kevlar 49/CE 3305 (material M1)
- Graphite-Epoxy AS-1/3501-5A (material M2)

For laminates of total thickness of 1mm with four sheets of individual thickness of 0.25mm, bending stiffness matrix D has the following form [15]:

$$\begin{aligned} D_{ij} &= \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})^k (h_k^3 - h_{k-1}^3) \\ &= \frac{1}{3} (\bar{Q}_{ij})_{\theta} (h_1^3 - h_0^3) + \frac{1}{3} (\bar{Q}_{ij})_{\theta} (h_2^3 - h_1^3) + \frac{1}{3} (\bar{Q}_{ij})_{\theta} (h_3^3 - h_2^3) + \frac{1}{3} (\bar{Q}_{ij})_{\theta} (h_4^3 - h_3^3) \\ &= \frac{1}{3} (\bar{Q}_{ij})_{\theta} [(-0,25)^3 - (-0,5)^3] + \frac{1}{3} (\bar{Q}_{ij})_{\theta} [(0)^3 - (-0,25)^3] + \frac{1}{3} (\bar{Q}_{ij})_{\theta} [(0,25)^3 - (0)^3] + \\ &\quad \frac{1}{3} (\bar{Q}_{ij})_{\theta} [(0,5)^3 - (0,25)^3] \end{aligned}$$

Based on the above expression and using the MATLAB software package, bending stiffness matrix for selected laminate schemes $\theta = 30^\circ$ and $\theta = 0^\circ$ are obtained.

$$[D] = \begin{bmatrix} 6,7954 & 0,3167 & 0,3990 \\ 0,3167 & 0,4800 & 0,1298 \\ 0,3990 & 0,1298 & 0,4841 \end{bmatrix} \quad [D] = \begin{bmatrix} 9,8649 & 0,3548 & 0,2588 \\ 0,3548 & 0,9093 & 0,0941 \\ 0,2588 & 0,0941 & 0,5000 \end{bmatrix}$$

Substituting the values of bending stiffness matrix in the previously set equations we obtain values of non-dimensional critical force for three types of buckling.

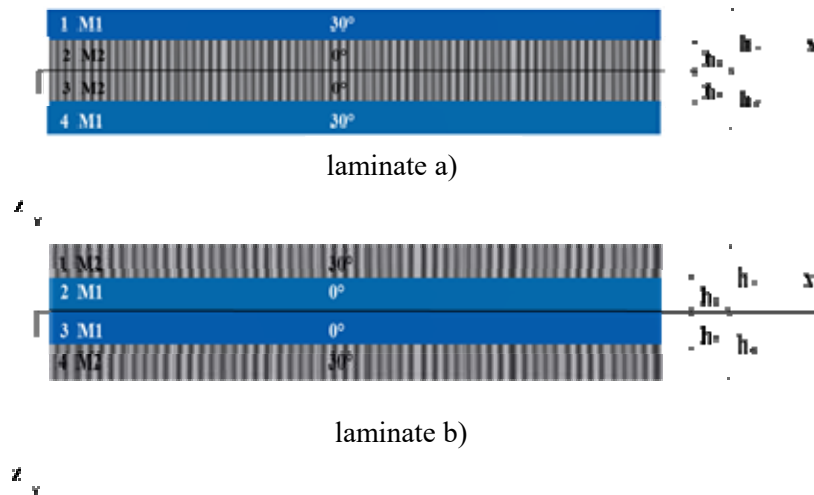


Figure 6. Schematic layout of composite laminate.

Based on equations (13), (16) and (18), in this section follows analysis of impact of aspect ratio a/b or lamina positions on the non-dimensional buckling load. Nondimensional buckling load is calculated for the value of Winkler modulus (k_w) which took following values: 10N/m^3 , 100N/m^3 and 1000N/m^3 , while the shear modulus parameter (k_G) took following values: 1N/m , 10N/m and 100N/m . The number of half waves was $m = 1$, $n = 1$, while the compression ratio was $\delta=0,5$. The thickness of one composite plate is $h = 0,25 \text{ mm}$, while the length takes values within $a=0-0,6\text{m}$ range and width takes value $b=0,3$.

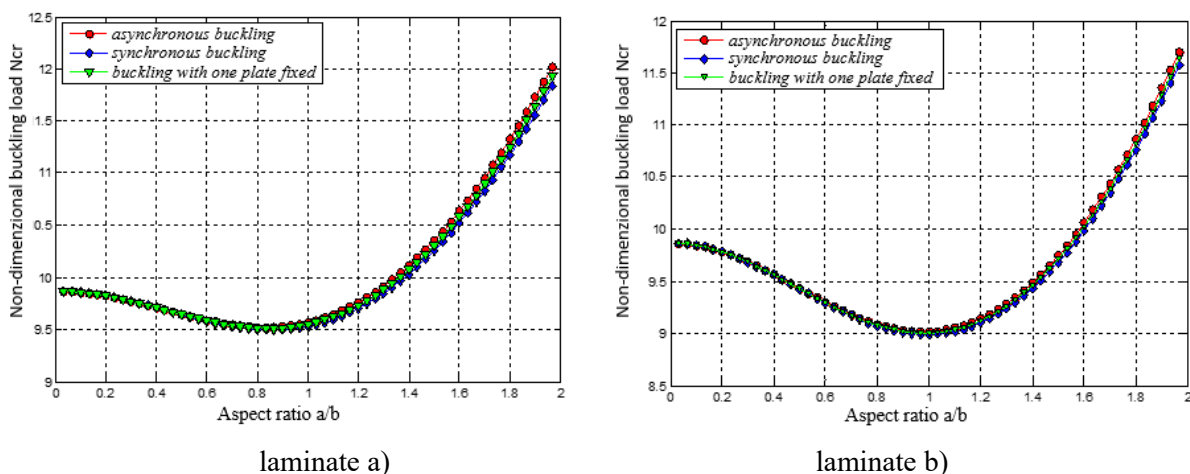
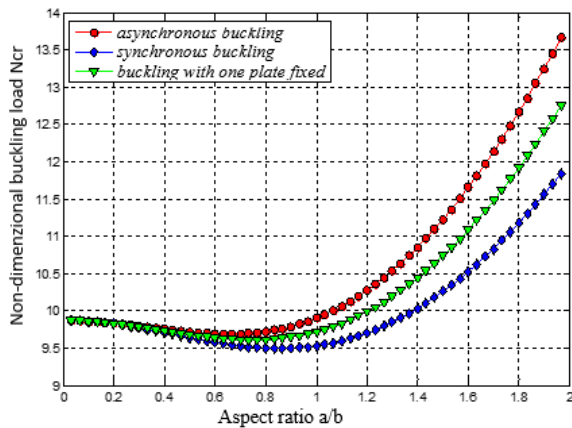
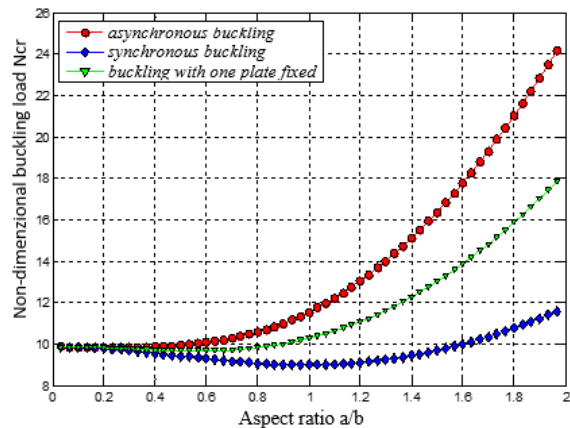


Figure 7. Effect of aspect ratio a/b on non-dimensional buckling load $k_w=10\text{N/m}^3$, $k_G=1\text{N/m}$.

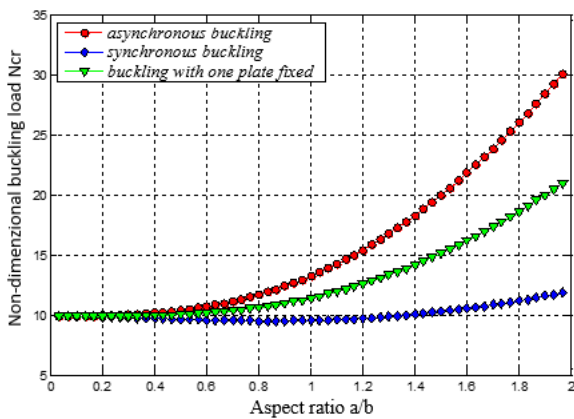


laminate a)

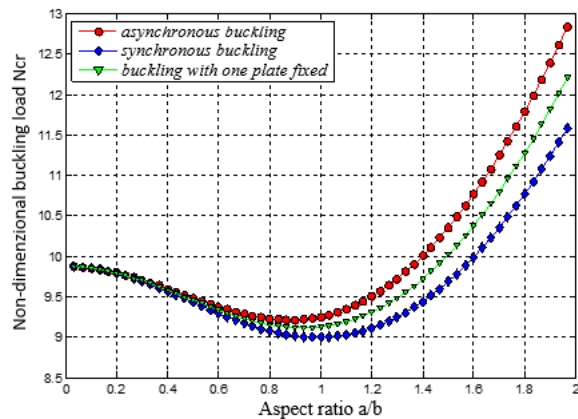


laminate b)

Figure 8. Effect of aspect ratio a/b on non-dimensional buckling load $k_w=100N/m^3, k_G=10N/m$.



laminate a)



laminate b)

Figure 9. Effect of aspect ratio a/b on non-dimensional buckling load $k_w=1000N/m^3, k_G=100N/m$.

The influence of Winkler modulus parameter k_w , which represents the influence of polymer matrix on the two composite plates, can be seen in figures 7-9. It can be concluded that higher values of Winkler modulus parameter increase the nondimensional buckling load the same for all three buckling types. By analyzing all the previous diagrams it can be concluded that the laminate types a) and b) should be avoided in the required boundary conditions and parameter values. For aspect ratio $a/b=0-0.8$, the curves are very close together for all three buckling types. For value of aspect ratio $a/b=0.8-1$ a very small value of the non-dimensional critical force leads to the deformation of the composite plates and the occurrence of instability of the system.

By increasing the relationship of dimensions $a/b=0.8-2$, the non-dimenzional critical force value is constantly increasing. It is obvious that the type of material of individual laminates has a great influence on the value of critical buckling force and should be taken into account. It can be concluded that in the case of laminate a) and b) we have behavior similar to the behavior of the isotropic and orthotropic plates.

5. Conclusion

Based on CLPT, in this paper was analyzed influence of aspect ratio a/b , fiber orientation angle and lamina positions on the non-dimensional buckling load on biaxial compressed composite plates embedded in elastic medium. In this paper, there are analytical expressions for non-dimensional buckling load for three characteristic cases of buckling of simply supported composite plates. It was shown that the increase of Winkler modulus parameter and shear modulus parameter also increases the nondimensional buckling load for all three characteristic buckling cases. The layout of the lamina inside the laminate has a significant effect as well as fiber orientation angle on the buckling and can not be ignored. It has been shown that with the change of lamina positions in the laminate, the value of the non-dimensioning critical load is changed for all three characteristic buckling cases. Laminates have different minimum and maximum values of non-dimensional critical force at the same value of aspect ratio. For asynchronous type of buckling non-dimensional critical buckling force has the highest value in both schemas of laminate plates.

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