

Dynamic Analysis of the Crane with Loading-unloading Trolley on the Slewing Platform

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Dynamic processes of the cranes with loading-unloading trolley on slewing platform are very complex and require certain simplifications, although efforts should be made to theoretically involve widest range of impacts, with respect to degree of importance of individual impacts. The study of the dynamic behavior of the cranes is very important in terms of precise determination of stress state, as well as the other characteristics of the elements and supporting structures. Dynamic loads of the cranes occur during periods of non-stationary operating regimes of any of its mechanisms. In this paper, we obtain the differential equations of motion of the cranes for some specific cases of movement, based on general multi-mass model of the crane with loading-unloading trolley on slewing platform.

Keywords: Rotary crane, dynamic loading, Equations of motion

1. INTRODUCTION

In order to include all loads when designing and modernization of the cranes with loading-unloading trolley on slewing platform, general method for dynamic analysis of their metal structures is proposed.

Problem of dynamic behaviour of cranes was considered in [1÷5]. Oscillations in the metal structure, which is a system with an infinite number of degrees of freedom, will be discussed as oscillating of elastic system with a finite number of degrees of freedom. Because of that the mass of the construction is replaced by one or several reduced masses, whereby the system must have a minimum number of degrees of freedom. Replacing the existing masses of construction with the reduced masses comes from the assumption of dynamic equivalence of both systems.

The rigidity of the elements of the mechanism is considerably higher than the rigidity of metal construction. Thus the flexibility of solid transfer components can be neglected, without reducing accuracy of calculations of the metal construction. This is confirmed by the experimental data of the tensometer testing of the hoist and movement mechanisms. When considering the mathematical model own structural damping is neglected because of its relatively small influence.

2. DYNAMIC MODEL OF CRANE

Based on analysis of construction of cranes with loading-unloading trolley on slewing platform, a mathematical model of their metal construction as elastic dynamic system is made (Fig. 1). We will consider the metal construction of cranes as an eight-mass system with thirteen generalized coordinates. The given mathematical model is general for all types of cranes with loading-unloading trolley on slewing platform and it corresponds to the real working conditions of metal construction, because it includes all elements of the system elasticity and has a minimal number of degrees of freedom.

We will consider the case of simultaneous operation of all the basic mechanisms of crane: the movement of the bridge, trolley movement, lifting and

rotation. System of equations of motion of the metal structure in general form is obtained, and it can be defined practically all possible cases of the crane operation.

3. MATHEMATICAL FORMULATION OF THE OSCILLATIONS

As shown in Fig. 1, there are two coordinate systems on the mathematical model. One of them is the $O_n X_n Y_n Z_n$ with the coordinate origin in O_n , and it is fixed. The other coordinate system is $OXYZ$ with the coordinate origin in O , and it moves together with the masses m_7 . Origin of the coordinate system $OXYZ$ is chosen as the middle of the left beam of the bridge in the position of equilibrium of elastic system. The axis X is directed horizontally along the span of the bridge to the side of the right beam of the bridge. The axis Z is directed horizontally along the beam of the bridge, and the axis Y is directed vertically on the underside of the crane.

The reduced masses are:

- m_1 - mass of the left beam of the bridge (without beam ends), presented in the middle of the bridge;
- m_2 - mass of the right beam of the bridge (without beam ends), presented in the middle of the bridge;
- m_3 - part of the mass of the gripping device, related to the left half of the bridge;
- m_4 - part of the mass of the gripping device, related to the right half of the bridge;
- m_5 - part of the mass of the gripping device, which is reduced to the upper end of the gripping devices;
- m_6 - mass of the payload and part of the mass of horizontal overhanging beam;
- m_7 - beam ends mass and part of the the reduced mass of the bridge.

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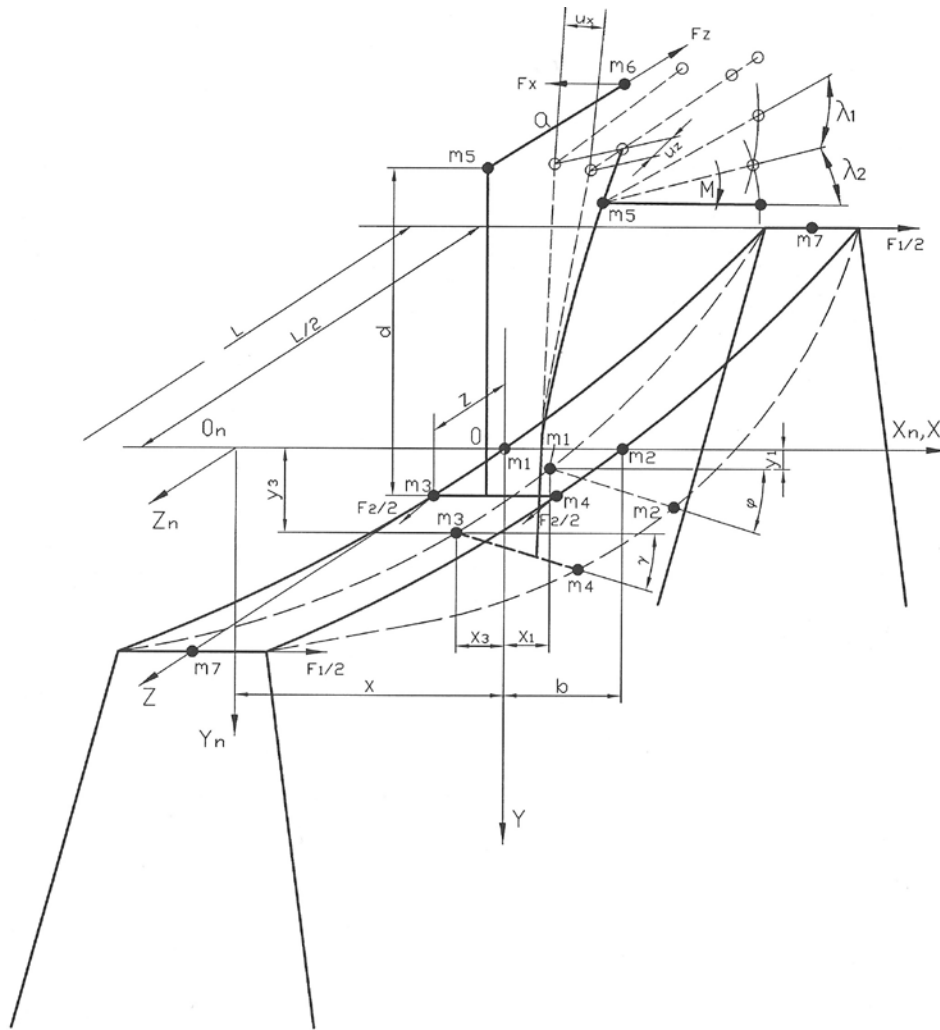


Figure 1: The multi-mass model of a crane with loading-unloading trolley on slewing platform

Generalized coordinates are:

- x - horizontal movement of point O along axis X from fixed point O_n ;
- x_1 - horizontal movement of mass m_1 along axis X from point O ;
- y_1 - vertical movement of mass m_1 along axis Y from point O ;
- φ - angle between the reduced masses m_1 and m_2 ;
- x_3 - horizontal movement of mass m_3 along axis X from point O ;
- y_3 - vertical movement of mass m_3 along axis Y from point O ;
- z - horizontal movement of mass m_3 along axis Z from point O ;
- γ - rotation angle of trolley;
- d - vertical movement of masses m_5 and m_6 ;
- u_x - movement of the upper end of the vertical overhanging beam along axis X ;
- u_z - movement of the upper end of the vertical overhanging beam along axis Z ;

- λ_2 - rotation angle of the horizontal overhanging beam in the swing;
- λ_1 - rotation angle of the horizontal overhanging beam around vertical overhanging beam.

The generalised non-conservative forces:

- F_1 - force of the the bridge movement mechanism;
- F_2 - force of the the trolley movement mechanism;
- M - moment on the horizontal overhanging beam;
- F_x i F_z - force components at the end of the horizontal overhanging beam.

For deriving the differential equations of motion, the second-order Lagrange equations of the following form were used. Basis for the mathematical model shown in Fig. 1,:

$$\frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{q}} \right) - \frac{\partial E_k}{\partial q} + \frac{\partial E_p}{\partial q} = Q_q \quad (1)$$

where

$q = f(x, x_1, y, \varphi, x_3, y_3, z, \gamma, d, u_x, u_z, \lambda_2, \lambda_1)$ -independent generalised coordinates;

E_k i E_p - kinetic and potential energy of the elastic system [6].

$$E_k = \frac{1}{2}a_{00}\dot{x}^2 + a_{01}\dot{x}\dot{x}_1 + a_{04}\dot{x}\dot{x}_3 + a_{07}\dot{x}\dot{\gamma} + a_{09}\dot{x}\dot{u}_x + a_{0,12}\dot{x}\dot{\lambda}_1 + \frac{1}{2}a_{11}\dot{x}_1^2 + \frac{1}{2}a_{22}\dot{y}_1^2 + a_{23}\dot{y}_1\dot{\varphi} + \frac{1}{2}a_{33}\dot{\varphi}^2 + \frac{1}{2}a_{44}\dot{x}_3^2 + a_{47}\dot{x}_3\dot{\gamma} + a_{49}\dot{x}_3\dot{u}_x + a_{4,12}\dot{x}_3\dot{\lambda}_1 + \frac{1}{2}a_{55}\dot{y}_3^2 + a_{57}\dot{y}_3\dot{\gamma} + a_{58}\dot{y}_3\dot{d} + a_{5,11}\dot{y}_3\dot{\lambda}_2 + \frac{1}{2}a_{66}\dot{z}^2 + a_{6,10}\dot{z}\dot{u}_z + a_{6,12}\dot{z}\dot{\gamma} + \frac{1}{2}a_{77}\dot{\gamma}^2 + a_{78}\dot{\gamma}\dot{d} + a_{79}\dot{\gamma}\dot{u}_z + a_{7,11}\dot{\gamma}\dot{\lambda}_2 + a_{7,12}\dot{\gamma}\dot{\lambda}_1 + \frac{1}{2}a_{88}\dot{d}^2 + a_{8,11}\dot{d}\dot{\lambda}_2 + \frac{1}{2}a_{99}\dot{u}_x^2 + a_{9,12}\dot{u}_x\dot{\lambda}_1 + \frac{1}{2}a_{10,10}\dot{u}_z^2 + a_{10,12}\dot{u}_z\dot{\lambda}_1 + \frac{1}{2}a_{11,11}\dot{\lambda}_2^2 + \frac{1}{2}a_{12,12}\dot{\lambda}_1^2.$$

$$E_p = \frac{1}{2}c_{11}x_1^2 + c_{14}x_1x_3 + \frac{1}{2}c_{22}y_1^2 + c_{23}y_1\varphi + c_{25}y_1y_3 + c_{27}y_1\varphi + \frac{1}{2}c_{33}\varphi^2 + c_{35}\varphi y_3 + c_{37}\varphi\gamma + \frac{1}{2}c_{44}x_3^2 + \frac{1}{2}c_{55}y_3^2 + c_{57}y_3\gamma + \frac{1}{2}c_{77}\gamma^2 + \frac{1}{2}c_{88}d^2 + \frac{1}{2}c_{99}u_x^2 + \frac{1}{2}c_{10,10}u_z^2 + \frac{1}{2}c_{12,12}\lambda_1^2.$$

Generalized forces are obtained using virtual displacements:

- 1) $Q_x = F_1 + F_x + m_6 a \cos \lambda_1 \ddot{\lambda}_1$;
- 2) $Q_{x1} = 0$;
- 3) $Q_{y1} = 0$;
- 4) $Q_\varphi = 0$;
- 5) $Q_{x3} = F_x + m_6 a \cos \lambda_1 \ddot{\lambda}_1$;
- 6) $Q_{y3} = -(m_5 + m_6)\ddot{d}$;
- 7) $Q_z = F_2 + F_z + m_6 a \sin \lambda_1 \ddot{\lambda}_1$;
- 8) $Q_\gamma = -F_x d - \frac{1}{2}(m_5 + m_6)b\ddot{d} - m_6 a d \cos \lambda_1 \ddot{\lambda}_1$;
- 9) $Q_d = -(m_5 + m_6)\ddot{d}$;
- 10) $Q_{u_x} = -F_x - m_6 a \cos \lambda_1 \ddot{\lambda}_1$;
- 11) $Q_{u_z} = F_z + m_6 a \sin \lambda_1 \ddot{\lambda}_1$;
- 12) $Q_{\lambda_2} = M + m_6 a \ddot{d}$;
- 13) $Q_{\lambda_1} = -F_x a - (I_5 + m_6 a^2)\ddot{\lambda}_1$.

Replacing concrete data in the (1) gives system of equations (2):

- 1) $a_{00}\ddot{x} + a_{01}\ddot{x}_3 - a_{04}\ddot{x}_3 + a_{07}\ddot{\gamma} + a_{09}\ddot{u}_x + a_{012}\ddot{\lambda}_1 = Q_x$;
- 2) $a_{10}\ddot{x} + a_{11}\ddot{x}_1 + c_{11}x_1 + c_{14}x_3 = 0$
- 3) $a_2\ddot{y}_1 + a_{23}\ddot{\varphi} + c_{22}y_1 + c_{23}\varphi + c_{25}y_3 + c_{27}\gamma = 0$;
- 4) $a_{32}\ddot{y}_1 + a_{33}\ddot{\varphi} + c_{32}y_1 + c_{33}\varphi + c_{35}y_3 + c_{37}\gamma = 0$;

- 5) $a_{40}\ddot{x} + a_{44}\ddot{x}_3 + a_{47}\dot{x}\dot{\gamma} + a_{49}u_x + a_{4,12}\ddot{\lambda}_1 + c_{41}x_1 + c_{44}x_3 = Q_{x3}$;
- 6) $a_{55}\ddot{y}_3 + a_{57}\ddot{\gamma} + a_{58}d_1 + a_{5,11}\ddot{\lambda}_2 + c_{52}y_1 + c_{53}\varphi + c_{55}y_3 + c_{57}\gamma = Q_{y3}$;
- 7) $a_{66}\ddot{z} + a_{6,10}\ddot{u}_z + a_{6,12}\ddot{\lambda}_1 = Q_z$;
- 8) $a_{70}\ddot{x} + a_{74}\ddot{x}_3 + a_{75}\ddot{y}_3 + a_{77}\ddot{\gamma} + a_{78}\ddot{d}_1 + a_{79}\ddot{u}_z + a_{7,11}\ddot{\lambda}_2 + a_{7,12}\ddot{\lambda}_1 + c_{72}y_1 + c_{73}\varphi + c_{75}y_3 + c_{77}\gamma = Q_\gamma$;
- 9) $a_{85}\ddot{y}_3 + a_{87}\ddot{\gamma} + a_{88}\ddot{d}_1 + a_{8,11}\ddot{\lambda}_2 + c_{88}d = Q_d$;
- 10) $a_{90}\ddot{x} + a_{94}\ddot{x}_3 + a_{97}\ddot{\gamma} + a_{99}\ddot{u}_x + a_{9,12}\ddot{\lambda}_1 + c_{99}u_x = Q_{u_x}$;
- 11) $a_{10,6}\ddot{z} + a_{10,10}\ddot{u}_z + a_{10,12}\ddot{\lambda}_1 + c_{10,10}u_z = Q_{u_z}$;
- 12) $a_{11,5}\ddot{y}_3 + a_{11,7}\ddot{\gamma} + a_{11,8}\ddot{d} + a_{11,11}\lambda_2 = Q_{\lambda_2}$;
- 13) $a_{12,0}\ddot{x} + a_{12,4}\ddot{x}_3 + a_{12,6}\ddot{z} + a_{12,7}\ddot{\gamma} + a_{12,9}\ddot{u}_x + a_{12,10}\ddot{u}_z + a_{12,12}\ddot{\lambda}_1 + c_{12,12}\lambda_1 = Q_{\lambda_1}$.

Where

a_{ik} - mass coefficients;

c_{ik} - stiffness coefficients;

i - number of equations;

k - number of independent variable.

- 1) $a_{00} = M$; $a_{01} = m_1 + m_2$;
 $a_{04} = m_3 + m_4 + m_5 + m_6$; $a_{07} = -(m_5 + m_6)d$;
 $a_{012} = -m_6 a \cos \alpha$.
- 2) $a_{10} = a_{01}$; $a_{11} = m_1 + m_2$; $c_{11} = 2c_{1r}$;
 $c_{14} = c_{4r} + c_{2r}$;
- 3) $a_{22} = m_1 + m_2$; $a_{23} = m_2 b$; $c_{22} = 2c_1$;
 $c_{23} = c_1 b$; $c_{25} = c_4 + c_2$; $c_{27} = \frac{1}{2}(c_4 + c_2)b$;
- 4) $a_{32} = a_{23}$; $a_{33} = m_2 b^2$; $c_{32} = c_{23}$; $c_{33} = c_1 b^2$;
 $c_{35} = \frac{1}{2}(c_4 + c_2)b$; $c_{37} = \frac{1}{2}(c_4 + c_6)b$;
- 5) $a_{40} = a_{04}$; $a_{44} = m_3 + m_4 + m_5 + m_6$;
 $a_{47} = -(m_5 + m_6)d$; $a_{4,12} = -m_6 a \cos \lambda_1$;
 $a_{49} = -(m_5 + m_6)$; $c_{41} = c_{14}$; $c_{44} = 2c_{3r}$;
- 6) $a_{55} = m_3 + m_4 + m_5 + m_6$;
 $a_{57} = \frac{1}{2}(2m_4 + m_5 + m_6)b$; $a_{58} = m_5 + m_6$;

$$\begin{aligned}
 & a_{5,11} = -m_6 a; \quad c_{52} = c_{25} \quad c_{53} = c_{35}; \quad c_{55} = 2c_3; \\
 & c_{57} = 2c_3 b; \\
 7) & \quad a_{65} = m_3 + m_4 + m_5 + m_6; \quad a_{6,10} = m_5 + m_6; \\
 & \quad a_{6,12} = -m_6 a \sin \lambda_1; \\
 8) & \quad a_{70} = a_{74} = a_{07}; \quad a_{75} = a_{57}; \\
 & \quad a_{77} = \left(4m_4 + m_5 + m_6\right) \frac{b^2}{4} + (m_5 + m_6) d^2; \\
 & \quad a_{78} = \frac{1}{2} (m_5 + m_6) b; \quad a_{79} = (m_5 + m_6) d; \\
 & \quad a_{7,11} = -\frac{1}{2} m_6 a b; \quad a_{7,12} = -\frac{1}{2} m_6 a l \cos \lambda_1; \\
 & \quad c_{72} = c_{27}; \quad c_{73} = c_{37}; \quad c_{75} = c_{57}; \quad c_{77} = c_3 b^2; \\
 9) & \quad a_{85} = a_{88} = a_{58}; \quad a_{87} = a_{78}; \quad a_{8,11} = -m_6 a; \\
 & \quad c_{88} = \frac{EF}{d}; \\
 10) & \quad a_{90} = a_{94} = a_{09}; \quad a_{97} = a_{79}; \quad a_{99} = m_5 + m_6; \\
 & \quad a_{9,12} = -m a \cos \lambda_1; \quad c_{99} = \frac{3EI_z}{d^3}; \\
 11) & \quad a_{10,6} = a_{10,10} = a_{6,10}; \quad a_{10,12} = -m_6 a \sin \lambda_1; \\
 & \quad c_{10,10} = \frac{3EI_x}{d^3}; \\
 12) & \quad a_{11,5} = a_{11,8} = a_{5,11}; \quad a_{11,7} = a_{7,11}; \\
 & \quad a_{11,11} = m_6 a^2; \\
 13) & \quad a_{12,0} = a_{12,4} = a_{0,12}; \quad a_{12,6} = a_{6,12}; \quad a_{12,7} = a_{7,12}; \\
 & \quad a_{12,12} = I_5 + m_6 a^2; \quad a_{12,9} = a_{9,12}; \quad a_{12,10} = a_{10,12}; \\
 & \quad c_{12,12} = \frac{GI_p}{d} \cdot gf
 \end{aligned}$$

4. THE EQUATIONS OF MOTION OF THE TROLLEY

The multi-mass model of a crane with loading-unloading trolley on slewing platform is shown in figure 2. The assumption is that the trolley is loaded and it is in the middle of the bridge, transverse to the axis of the bridge. In that case we have:

$$\begin{aligned}
 x &= 0; \quad \ddot{x} = 0; \\
 d &= const; \quad \ddot{d} = 0; \\
 x_1 &= 0; \quad \ddot{x}_1 = 0; \\
 u_x &= 0; \quad \ddot{u}_x = 0; \\
 x_3 &= 0; \quad \ddot{x}_3 = 0;
 \end{aligned}$$

$$\begin{aligned}
 \lambda_2 &= 0; \quad \ddot{\lambda}_2 = 0; \\
 y_3 &\approx y_1; \quad \ddot{y}_3 \approx \ddot{y}_1; \\
 \lambda_1 &= \pi/2; \quad \ddot{\lambda}_1 = 0; \\
 \varphi &\approx \gamma; \quad \ddot{\varphi} \approx \ddot{\gamma}; \\
 F_1 &= M = F_x = 0.
 \end{aligned}$$

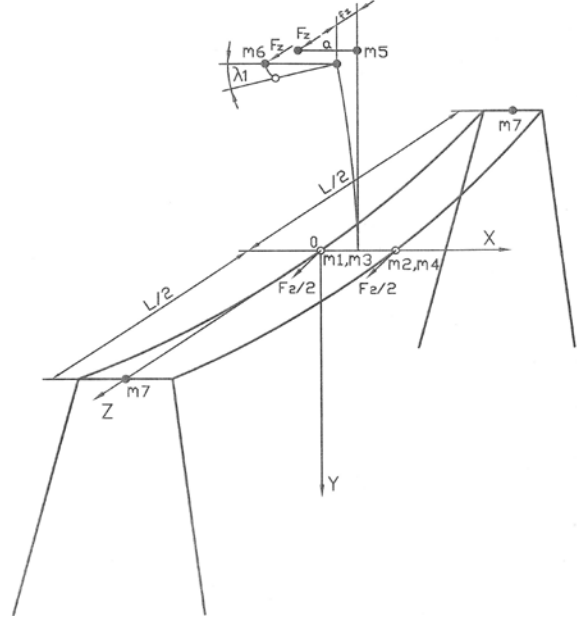


Figure 2: The multi-mass model of a crane in case of loaded trolley movement

The system of equations (2) becomes:

$$\begin{aligned}
 a_{66} \ddot{z} + a_{6,10} \ddot{u}_z + a_{6,12} \ddot{\lambda}_1 &= F_2 + F_z; \\
 a_{10,6} \ddot{z} + a_{10,10} \ddot{u}_z + a_{10,12} \ddot{\lambda}_1 + c_{10,10} u_z &= F_z; \\
 a_{12,6} \ddot{z} + a_{12,10} \ddot{u}_z + a_{12,12} \ddot{\lambda}_1 + c_{12,12} \lambda_1 &= -F_z a.
 \end{aligned}$$

Elimination of members which contains term \ddot{z} gives following system of equations (3):

$$\begin{aligned}
 a_{1f} \ddot{u}_z + a_{1\alpha} \ddot{\lambda}_1 + c_f u_z &= a_{1Q} \\
 a_{2f} \ddot{u}_z + a_{2\alpha} \ddot{\lambda}_1 + c_\alpha \lambda_1 &= a_{2Q}
 \end{aligned} \tag{3}$$

Where

$$\begin{aligned}
 a_{1\alpha} &= a_{66} a_{10,12} - a_{10,12} - a_{10,6} a_{6,10}; \\
 c_f &= a_{66} c_{10,10}; \\
 a_{1Q} &= -a_{10,6} F_2 + (a_{66} - a_{10,6}) F_z; \\
 a_{2f} &= a_{66} a_{12,10} - a_{12,6} a_{6,10}; \\
 a_{2\alpha} &= a_{66} a_{12,12} - a_{12,6} a_{6,12}; \\
 c_f &= a_{66} c_{12,12}; \\
 a_{2Q} &= -a_{12,6} F_2 - (a_{12,6} - a_{66} a) F_z.
 \end{aligned}$$

Non-conservative forces F_2 and F_z acting on the system.

Variation I. $F_2(t) = N_2 e^{-n_2 t}$; $F_z = const$.

The system of equations (3) takes the form:

$$\begin{aligned} a_{1f}\ddot{u}_z + a_{1\alpha}\ddot{\lambda}_1 + c_f u_z &= -a_{10.6}N_2 e^{-n_2 t} + (a_{66} - a_{10.6})F_z; \\ a_{2f}\ddot{u}_z + a_{2\alpha}\ddot{\lambda}_1 + c_\alpha \lambda_1 &= -a_{12.6}N_2 e^{-n_2 t} - (a_{12.6} - a_{66}a)F_z. \end{aligned}$$

The general solution of this system of equations is:

$$u_z = A_{10}^{(10)} \sin(\omega_{10}t + \delta_{10}) + A_{10}^{(12)} \sin(\omega_{12}t + \delta_{12}) + C_{10}e^{-n_2 t} + D_{10};$$

$$\lambda_1 = A_{12}^{(10)} \sin(\omega_{10}t + \delta_{10}) + A_{12}^{(12)} \sin(\omega_{12}t + \delta_{12}) + C_{12}e^{-n_2 t} + D_{12}.$$

Where

$$C_{10} = \frac{\begin{vmatrix} -a_{10.6}N_2 & a_{1\alpha}n_2^2 \\ -a_{12.6}N_2 & a_{2\alpha}n_2^2 - c_\alpha \end{vmatrix}}{\begin{vmatrix} a_{1f}n_2^2 + c_f & a_{1\alpha}n_2^2 \\ a_{2f}n_2^2 & a_{2\alpha}n_2^2 + c_\alpha \end{vmatrix}} = \frac{\Delta_{10}}{\Delta};$$

$$C_{12} = \frac{\Delta_{12}}{\Delta}.$$

Δ - determinant of the system of differential equations;

Δ_{10} and Δ_{12} - determinants where first and second column were replaced with the values on the right side of above listed equations.

$$D_{10} = \frac{(a_{66} - a_{10.6})F_z}{c_f};$$

$$D_{12} = \frac{-(a_{12.6} - a_{66}a)F_z}{c_\alpha}.$$

Variant II. $F_2 = N_2 = const$; $F_z = const$.

The system of equations (3) takes the form:

$$\begin{aligned} a_{1f}\ddot{u}_z + a_{1\alpha}\ddot{\lambda}_1 + c_f u_z &= -a_{10.6}N_2 + (a_{66} - a_{10.6})F_z; \\ a_{2f}\ddot{u}_z + a_{2\alpha}\ddot{\lambda}_1 + c_\alpha \lambda_1 &= -a_{12.6}N_2 - (a_{12.6} - a_{66}a)F_z. \end{aligned}$$

The general solution of this system of equations is:

$$u_z = A_{10}^{(10)} \sin(\omega_{10}t + \delta_{10}) + A_{10}^{(12)} \sin(\omega_{12}t + \delta_{12}) + D'_{10}$$

$$\lambda_1 = A_{12}^{(10)} \sin(\omega_{10}t + \delta_{10}) + A_{12}^{(12)} \sin(\omega_{12}t + \delta_{12}) + D'_{12}$$

Where

$$D'_{10} = \frac{-a_{10.6}N_2 + (a_{66} - a_{10.6})F_z}{c_f};$$

$$D'_{12} = \frac{a_{12.6}N_2 - (a_{12.6} - a_{66}a)F_z}{c_\alpha}.$$

Dynamic loads:

$$P_{vz} = u_z c_v;$$

$$M_{zu} = \lambda_1 c_{zu},$$

Where

P_{vz} - load on the upper end of the gripping devices along axis Z;

c_v - flexural stiffness;

M_{zu} - moment on the upper end of the gripping devices;

c_{zu} - torsional stiffness.

We will consider the case of the approximate solution of equations of motion during functioning of trolley mechanism, under following assumption

$$F_2 = N_2 = const; F_z = const.$$

I case. Approximate determination of dynamic loads which acting on the horizontal plane on the upper end of the gripping devices along axis Z. We assume that the trolley is absolutely rigid in torsion. Differential equation of motion is:

$$a_{1f}\ddot{u}_z + c_f u_z = -a_{10.6}N_2 + (a_{66} - a_{10.6})F_z,$$

and angular frequency of oscillation

$$\omega_f = \sqrt{\frac{c_f}{a_{1f}}}.$$

Solution of differential equation:

$$u_z = A_f \sin(\omega_f t + \delta'_f) + A_{10}^{(12)} \sin + D'_{10}$$

Horizontal dynamic load which acting on the upper end of gripping devices:

$$P_{vz} = u_z c_v.$$

II case. Approximate determination of the moment which acting at the upper end of gripping devices. We assume that the trolley is absolutely rigid in bending. Differential equation of motion is:

$$a_{2\alpha}\ddot{\lambda}_1 + c_\alpha \lambda_1 = -a_{12.6}N_2 - (a_{12.6} - a_{66}a)F_z.$$

and angular frequency of oscillation

$$\omega_\alpha = \sqrt{\frac{c_\alpha}{a_{2f}}}.$$

Solution of differential equation:

$$\lambda_1 = A_\alpha \sin(\omega_\alpha t + \delta'_\alpha) + D'_{12}.$$

Moment which acting on the upper end of gripping devices:

$$M_{zu} = \lambda_1 c_{zu}.$$

The multi-mass model of a crane in case of unloaded trolley movement is shown in figure 3.

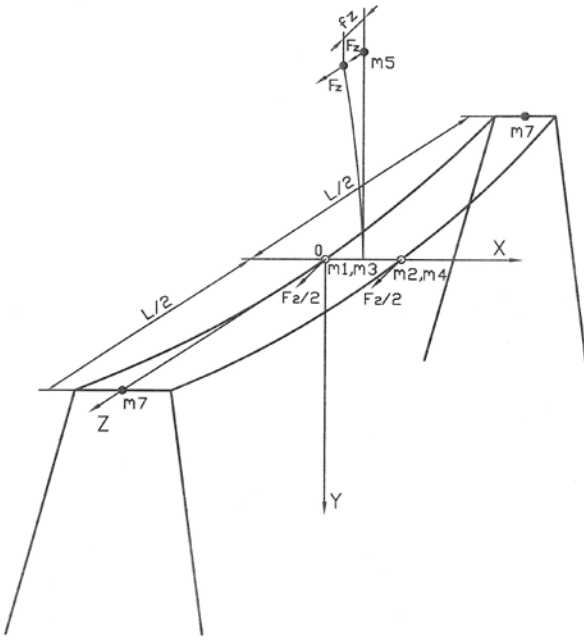


Figure 3: The multi-mass model of a crane in case of loaded trolley movement

Since the trolleys are not loaded, it is: $m_6 = 0$, $a = 0$, $\lambda_1 = \dot{\lambda}_1 = \ddot{\lambda}_1 = 0$. Now, we omitted members containing λ_1 , $\dot{\lambda}_1$ and $\ddot{\lambda}_1$. The system of equations (2) become:

$$a_{66}\ddot{z} + a_{6-10}\ddot{u}_z = F_2 + F_z;$$

$$a_{10-6}\ddot{z} + a_{10-10}\ddot{u}_z + c_{10-10}u_z = F_z.$$

If we eliminate members which contains \ddot{z} we obtained the equation:

$$(a_{66}a_{10-10} - a_{10-6}a_{6-10})\ddot{u}_z + a_{66}c_{10-10}u_z = a_{10-6}F_2 + (a_{66} - a_{10-6})F_z. \quad (4)$$

Or

$$a_{1f}\ddot{u}_z + c_f u_z = a_{10}.$$

Angular frequency of oscillation is:

$$\omega_{10} = \sqrt{\frac{c_f}{a_{1f}}}.$$

Variant I. $F_2(t) = N_2 e^{-n_2 t}$; $F_z = const$.

The equation (4) becomes:

$$a_{1f}\ddot{u}_z + c_f u_z = -a_{10-6}N_2 e^{-n_2 t} + (a_{66} - a_{10-6})F_z.$$

The general solution of equation is:

$$u_z = A_{10} \sin(\omega_{10} t + \delta_{10}) + C_{10} e^{-n_2 t} + D_{10},$$

Where

$$C_{10} = \frac{a_{10-6}N_2}{a_{1f}n_2^2 + c_f}; \quad D_{10} = \frac{(a_{66} - a_{10-6})F_z}{c_f}.$$

Variant II. $F_2 = N_2 = const$; $F_z = const$.

The equation (4) becomes:

$$a_{1f}\ddot{u}_z + c_f u_z = -a_{10-6}N_2 + (a_{66} - a_{10-6})F_z.$$

The general solution of equation is:

$$u_z = A_{10} \sin(\omega_{10} t + \delta_{10}) + D_{10},$$

where

$$D_{10} = \frac{-a_{10-6}N_2 + (a_{66} - a_{10-6})F_z}{c_f}$$

5. CONCLUSION

In the paper, a model of a crane with loading-unloading trolley on the slewing platform with eight mass and thirteen degrees of freedom is presented. Lagrange equations were used to derive the equations of motion. The obtained system of equations (2) represents the movement of the metal construction of the crane with loading-unloading trolley on the slewing platform, which enables determination of the dynamic loads that acting on a metal structure in separate and simultaneous operation of all mechanisms of cranes in different periods of operating.

Equations of motion of trolley are obtained, in case that the trolley is placed in the middle of the bridge, transversely to the axis of the bridge. We considered the cases of loaded and unloaded trolley and based on that, the equations of dynamic loads and angular frequency, as essential elements of the dynamic stability of the system, were obtained.

6. REFERENCES

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